

1. [60 points] [losslessness & synthesis]

a) Find the conditions on the real constants a, b, c, d such that the following scattering function ($=1 \times 1$ matrix=reflection coefficient) is (passive and) lossless

$$S(s) = \frac{(s-a)(s-b)}{(s+c)(s+d)}$$

b) For all these lossless $S(s)$ of a), realize $S(s)$ by connecting two reflection coefficients, $S_1(s)$ and $S_2(s)$, to a 3-port circulator.

c) For all the lossless $S(s)$ of a) convert to $Y(s)$ and realize by the two Foster forms and the two Cauer forms.

d) For all the lossless $S(s)$ of a) realize via the Richards function choosing $k=1$.

e) Repeat parts c) and d) in the presence of complex constants under the condition that $a=c=b^*=d^*$.

2. [40 points] [Richards' functions]

If $f(s)$ is PR show that if any one of the following is PR then so are all of the rest:

$$g_1(s) = f(k) \frac{kf(k) - sf(s)}{kf(s) - sf(k)}$$

$$g_2(s) = f(k) \frac{kf(s) - sf(k)}{kf(k) - sf(s)}$$

a)

$$g_3(s) = \frac{1}{f(k)} \frac{[k/f(k)] - [s/f(s)]}{[k/f(s)] - [s/f(k)]}$$

$$g_4(s) = \frac{1}{f(k)} \frac{[k/f(s)] - [s/f(k)]}{[k/f(k)] - [s/f(s)]}$$

b) Assuming k real and $f(s)=y(s)$, an admittance, show which of the $g_i(s)$, $i=1, \dots, 4$, of part a) are admittances and which are impedances and give a circuit to realize each of the $g_i(s)$ in part a) as the load on a passive 2-port.