

## 1. [70 points] [semistate evaluation and design]

For the following semistate equations all the coefficient matrix entries are real valued scalars

$$\begin{bmatrix} 0 & e_{12} \\ 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} x$$

- a) Find the (scalar) transfer function,  $T(s)$ , and show that there are some coefficient values for which  $T(s)=s$  and another set of coefficients for which  $T(s)=1$ .
- b) Show that the following set of equations can have the same transfer function,  $T(s)$

$$\begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix} \frac{dX}{dt} = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix} X + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} X$$

- c) Writing these two equivalent semistate equations as  
 $edx/dt=ax+bu, y=cx$

and

$$EdX/dt=AX+Bu, y=CX$$

find the transformation pair,  $P$  and  $Q$  (both nonsingular),  $PaQ=A$ , to go between the two.

- d) In the case of  $T(s)=s$  with  $u=i$  (current) and  $y=v$  (voltage) draw an OTA-C circuit to give the semistate equations of b) above.

## 2. [30 points] [positive and bounded real conditions]

For each of the following functions state a) if it is positive real and b) if it is bounded real as well as c) if it is lossless.

a)  $f(s) = \frac{s(s^2+9)(s^2+25)}{(s^2+4)(s^2+16)}$ .

b)  $f(s) = \frac{as+b}{cs+d}$ ,  $a > b > c > d > 0$  otherwise freely chosen (real).

c)  $f(s) = \left(\frac{s-2}{s+2}\right) \left(\frac{s^2-4s+4}{s^2+4s+4}\right)$