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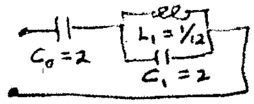
#1. $y(s) = \dfrac{s(s^2+6)}{s^2+3} \implies z(s) = \dfrac{1}{y(s)} = \dfrac{s^2+3}{s(s^2+6)}$

1st Foster: $z(s) = \dfrac{k_0}{s} + \dfrac{k_1 s}{s^2+6}$, $k_0 = s\,z(s)\Big|_{s=0} = \dfrac{s^2+3}{s^2+6}\Big|_{s=0} = \dfrac{3}{6} = \dfrac{1}{2}$

$k_1 = \dfrac{s^2+6}{s} \cdot z(s)\Big|_{s^2=-6} = \dfrac{s^2+3}{s^2}\Big|_{s^2=-6} = \dfrac{-6+3}{-6} = \dfrac{-3}{-6} = \dfrac{1}{2}$

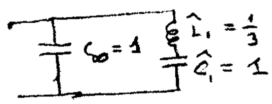
$= \dfrac{1/2}{s} + \dfrac{\frac{1}{2}s}{s^2+6} = \dfrac{1}{2s} + \dfrac{1}{2s + \frac{12}{s}}$

$\implies$ 

2nd Foster: $y(s) = k_\infty s + \dfrac{\hat{k}_1 s}{s^2+3}$, $k_\infty s = s^3/s^2 \implies k_\infty = 1$

$\hat{k}_1 = \dfrac{s^2+3}{s} y(s)\Big|_{s^2=-3} = s^2+6\Big|_{s^2=-3} = -3+6 = 3$

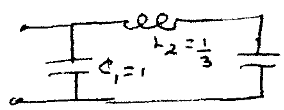
$= s + \dfrac{3s}{s^2+3} = s + \dfrac{1}{\frac{1}{3}s + \frac{1}{s}}$

$\implies$ 

1st Cauer:

$$\begin{array}{l} s^2+3 \,\big)\, s^3+6s \quad (s = y_1) \\ \quad s^3+3s \\ \hline \quad 3s \,\big)\, s^2+3 \quad (\tfrac{1}{3}s) \\ \qquad s^2 \\ \hline \qquad 3 \,\big)\, 3s \quad (s) \end{array}$$

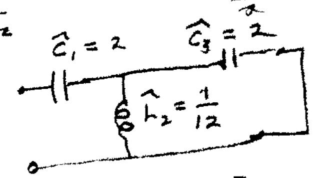
$\implies y = s + \dfrac{1}{\frac{1}{3}s + \frac{1}{s}}$

$\implies$  (same as 2nd Foster)

2nd Cauer

$$\begin{array}{l} 6s+s^3 \,\big)\, 3+s^2 \quad (\tfrac{1}{2s}) \\ \quad 3+\tfrac{1}{2}s^2 \\ \hline \quad \tfrac{1}{2}s^2 \,\big)\, 6s+s^3 \quad (12/s) \\ \qquad 6s \\ \hline \qquad s^3 \,\big)\, \tfrac{1}{2}s^2 \quad (\tfrac{1}{2s}) \end{array}$$

$\implies z = \dfrac{1}{2s} + \dfrac{1}{\frac{12}{s} + \frac{1}{\frac{1}{2s}}}$

$\implies$  (same as 1st Foster)

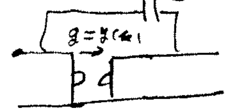
3 Richards' sections

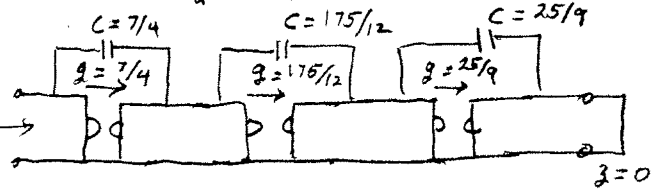
$y_1 = y(k)\left[\dfrac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}\right]$, $k = 1$

$y_1(s) = \dfrac{7}{4}\left[\dfrac{7/4 - s^2(s^2+6)/(s^2+3)}{s\left(\frac{s^2+6}{s^2+3}\right) - \frac{7}{4}s}\right] = \dfrac{7}{4}\left[\dfrac{-s^4 - \frac{17}{4}s^2 + \frac{21}{4}}{s\left(-\frac{3}{4}s^2 + \frac{3}{4}\right)}\right] = \dfrac{7}{4s}\left[\dfrac{-(s^2-1)(s^2+21/4)}{-\frac{3}{4}(s^2-1)}\right] = \dfrac{7}{3s}\left(s^2 + 21/4\right)$

$y_{11}(s) = \dfrac{7}{3}\cdot\dfrac{25}{4}\left[\dfrac{\frac{7}{3}\cdot\frac{25}{4} - \frac{7}{3}\left(s^2+\frac{21}{4}\right)}{\frac{7}{3s}\left(s^2+\frac{21}{4}\right) - \frac{7}{3}\cdot\frac{25}{4}s}\right] = \dfrac{7}{3}\cdot\dfrac{25}{4}s\left[\dfrac{-s^2+1}{-\frac{21}{4}s^2+\frac{21}{4}}\right] = \dfrac{7}{3}\cdot\dfrac{25}{4}s\cdot\dfrac{4}{21} = \dfrac{25}{9}s$

$y_{111}(s) = \dfrac{25}{9}\left[\dfrac{25/9 - \frac{25}{9}s^2}{\frac{25}{9}s - \frac{25}{9}s}\right] \implies z_{111} = 0$

Each section 

$y = \dfrac{s(s^2+6)}{s^2+3} \rightarrow$ 

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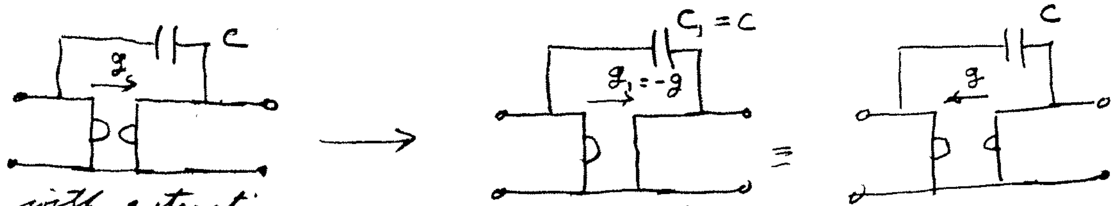
#2. a) as s is replaced by $-s$, $g_{old} = y(s) \to g_{new} = y(-s) = -y(s)$ so $g_{new} = -g_{old}$
g is replaced by its negative. For $C_{old} = \frac{y(s)}{s} \to C_{new} = \frac{y(-s)}{-s} = \frac{-y(s)}{-s} = C_{old}$

So the capacitor retains its old value and the gyrator is turned around



with extraction at $s = s_1$:
$g = y(s_1)$
$C = y(s_1)/s_1$

with extraction $s = -s_1$
$g_1 = y(s) = -y(s_1) = -g$
$C_1 = \frac{y(s)}{s} = \frac{-y(s_1)}{-s_1} = C$

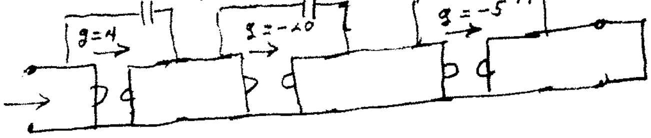
b) as each Richards' function extraction of an even part zero also extracts the negative of the zero, so each extraction removes 2 even part zeros. It also lowers the degree by one, so at a minimum $2\delta[y(s)]$ even part zeros are removed. In the case of lossless $y(s)$, there are an infinite number of even part zeros, so no maximum.

c) as the Richards' function lowers the degree of a rational $y(s)$ irrespective of it being PR it can be used for active synthesis (the C's in the sections will not all be >0!) For $y(s) = s^3+3s$, $y(s) = -y(-s)$ so any three real s's can be used; Choose $s_1 = s_2 = s_3 = 1$

$$y_r(s) = 4\left[\frac{4 - s^2(s^2+3)}{s(s^2+3) - 4s}\right] = \frac{4}{s}\left[\frac{-(s^2-1)(s^2+4)}{s^2-1}\right] = -\frac{4(s^2+4)}{s}$$

$$y_{rr}(s) = -20\left[\frac{-20-(-4(s^2+4))}{\frac{-4(s^2+4)}{s} - s(-20)}\right] = -20s\left[\frac{4-4s^2}{-16s^2+16}\right] = -5s$$

$$y_{rrr}(s) = -5\left[\frac{-5 - s(-5s)}{-5s - s(-5)}\right] = \frac{-25(s^2-1)}{0} \Rightarrow z_{rrr} = 0$$

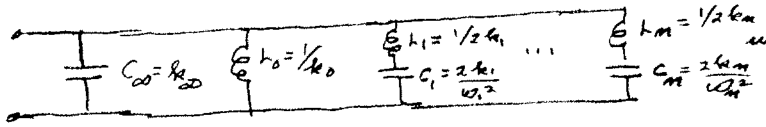


$y = s^3+3s \rightarrow$

#3. For an LC circuit we know if $y_{LC}(s)$ exists (i.e. $z_{LC} \neq 0$), by 2nd Foster,

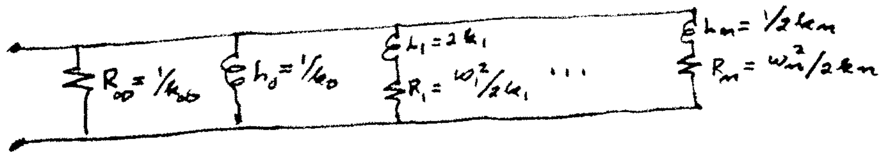
$$y_{LC}(s) = k_\infty s + \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} \qquad \text{all } k_i > 0,\ i = 0, \dots, n, \infty$$

in which case the circuit is



where $y_i = \dfrac{1}{\frac{s}{2k_i} + \frac{\omega_i^2}{2k_i s}}$, $i = 1, \dots, n$

Then for LR replace each C by $G = 1/R$. Then the "derived" RL circuit is



$y_i = \dfrac{1}{\frac{s}{2k_i} + \frac{\omega_i^2}{2k_i}}$

This has

$$y_{LR}(s) = k_\infty + \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i}{s+\omega_i^2}$$

If we set $s = p^2$ & multiply by p we get

$$p\,y_{LR}(p^2) = k_\infty p + \frac{k_0}{p} + \sum_{i=1}^{n} \frac{2k_i p}{p^2+\omega_i^2} = y_{LC}(p)$$

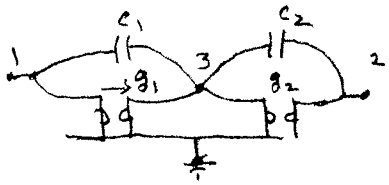
Letting p back to s, $s\,y_{LR}(s^2) = y_{LC}(s)$

necessity $\therefore$ $s\,y_{LR}(s^2) = -[-s\,y_{LR}((-s)^2)]$ ← necessarily true

sufficiency Since any PR $y(s)$ for which $s\,y(s^2)$ is LC it can be synthesized by the above circuit the condition is sufficient

$\therefore$ a PR $y(s)$ is for an LR circuit if and only if $s\,y(s^2)$ is lossless.

#4.



$$Y_{nodal}(s) = \begin{bmatrix} sC_1 & 0 & -sC_1+g_1 \\ 0 & sC_2 & -sC_2+g_2 \\ -sC_1-g_1 & -sC_2-g_2 & sC_1+sC_2 \end{bmatrix}$$

by inspection of components attached to the nodes.

We wish to eliminate node 3 $\Rightarrow i_3 = 0 \Rightarrow$

$$\begin{bmatrix} -sC_1-g_1 & -sC_2-g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -\begin{bmatrix} sC_1+sC_2 \end{bmatrix}\begin{bmatrix} v_3 \end{bmatrix}$$

$$\Rightarrow v_3 = \begin{bmatrix} \frac{sC_1+g_1}{s(C_1+C_2)} & \frac{sC_2+g_2}{s(C_1+C_2)} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\therefore Y_{2port} = \begin{bmatrix} sC_1 & 0 \\ 0 & sC_2 \end{bmatrix} + \begin{bmatrix} -sC_1+g_1 \\ -sC_2+g_2 \end{bmatrix} \begin{bmatrix} \frac{sC_1+g_1}{s(C_1+C_2)} & \frac{sC_2+g_2}{s(C_1+C_2)} \end{bmatrix}$$

$$s(C_1+C_2)\cdot y_{11} = sC_1\cdot s(C_1+C_2) + (-sC_1+g_1)(sC_1+g_1) = s^2C_1C_2 + g_1^2$$

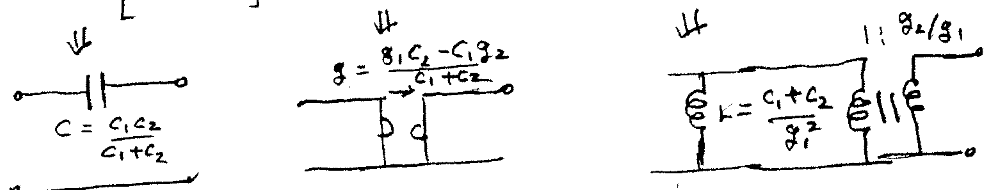
$$s(C_1+C_2)\cdot y_{12} = (-sC_1+g_1)(sC_2+g_2) = -s^2C_1C_2 + s(g_1C_2 - C_1g_2) + g_1g_2$$

$$s(C_1+C_2)\cdot y_{21} = (-sC_2+g_2)(sC_1+g_1) = -s^2C_1C_2 - s(g_1C_2 - C_1g_2) + g_1g_2$$

$$s(C_1+C_2)\cdot y_{22} = sC_2\cdot s(C_1+C_2) + (-sC_2+g_2)(sC_2+g_2) = s^2C_1C_2 + g_2^2$$

$$\therefore Y_{2port}(s) = \frac{sC_1C_2}{C_1+C_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{g_1C_2 - C_1g_2}{C_1+C_2}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \frac{g_1^2}{s(C_1+C_2)}\begin{bmatrix} 1 & g_2/g_1 \\ g_2/g_1 & (g_2/g_1)^2 \end{bmatrix}$$

$\Downarrow$ $\Downarrow$ $\Downarrow$



all in parallel

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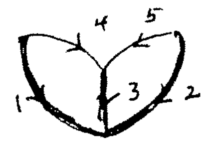
#5: a)



Cutset: $Q_3 = \begin{bmatrix}0\\0\\0\end{bmatrix} = \left[\begin{array}{ccc|cc}1&0&0&1&0\\0&1&0&0&1\\0&0&1&-1&-1\end{array}\right]\begin{bmatrix}i_1\\i_2\\i_3\\i_4\\i_5\end{bmatrix}_b$

$\Rightarrow C = \begin{bmatrix}1&0&0&1&0\\0&1&0&0&1\\0&0&1&1&1\end{bmatrix}$

Tieset: $Q_2 = \begin{bmatrix}0\\0\end{bmatrix} = \left[\begin{array}{ccc|cc}-1&0&1&1&0\\0&-1&1&0&1\end{array}\right]\begin{bmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{bmatrix}_b$

$\Rightarrow T = \begin{bmatrix}-1&0&1&1&0\\0&-1&1&0&1\end{bmatrix}$

b) $Av = Bi$ gives

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&0&g\\0&0&0&-g&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{bmatrix} = \begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&sL&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}i_1\\i_2\\i_3\\i_4\\i_5\end{bmatrix}$$

$v_b = v + e$, $e = \begin{bmatrix}V_1\\V_2\\0\\0\\0\end{bmatrix}$

$i_b = i$

use $v_b = C^T v_t = v + e \Rightarrow v = C^T v_t - e$, $\quad i_b = T^T i_\ell = i = \begin{bmatrix}-1&0\\0&-1\\1&1\\1&0\\0&1\end{bmatrix}\begin{bmatrix}i_4\\i_5\end{bmatrix}$

$$\therefore A C^T v_t - A e = \begin{bmatrix}1&&&&\\&1&&&\\&&1&&\\&&&0&g\\&&&-g&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\1&0&1\\0&1&1\end{bmatrix}v_t - \begin{bmatrix}1&&&&\\&1&&&\\&&1&&\\&&&0&g\\&&&-g&0\end{bmatrix}\begin{bmatrix}V_1\\V_2\\0\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&g&g\\-g&0&-g\end{bmatrix}v_t - \begin{bmatrix}V_1\\V_2\\0\\0\\0\end{bmatrix} \quad \& \quad B T^T i_\ell = \begin{bmatrix}0&&&&\\&0&&&\\&&sL&&\\&&&1&\\&&&&1\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\\1&1\\1&0\\0&1\end{bmatrix}i_\ell = \begin{bmatrix}0&0\\0&0\\sL&sL\\1&0\\0&1\end{bmatrix}i_\ell$$

$$\therefore [A C^T, -B T^T]\, x = \begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&-sL&-sL\\0&g&g&-1&0\\-g&0&-g&0&-1\end{bmatrix}x = \begin{bmatrix}1&0\\0&1\\0&0\\0&0\\0&0\end{bmatrix}u, \quad x = \begin{bmatrix}v_1\\v_2\\v_3\\i_4\\i_5\end{bmatrix} = \begin{bmatrix}v_t\\i_\ell\end{bmatrix}$$

$y = \begin{bmatrix}-I_1\\-I_2\end{bmatrix} = \begin{bmatrix}0&0&0&-1&0\\0&0&0&0&-1\end{bmatrix}x$ using $i_b = T^T i_\ell$

$\therefore$ semistate equations

$$\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&L&L\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\frac{dx}{dt} = \begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&g&g&-1&0\\-g&0&-g&0&-1\end{bmatrix}x + \begin{bmatrix}-1&0\\0&-1\\0&0\\0&0\\0&0\end{bmatrix}u$$

$$y = \begin{bmatrix}0&0&0&-1&0\\0&0&0&0&-1\end{bmatrix}x$$