Homework 7 solution

11/11/09
p. 1
RWN

use $y_L(s) = y(k)\cdot\dfrac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$

a) $$y_L(s) = \frac{k_1(k_1^2+16)}{k_1^2+9}\cdot\frac{k_1^2\left(\frac{k_1^2+16}{k_1^2+9}\right) - s^2\left(\frac{s^2+16}{s^2+9}\right)}{k_1 s\left(\frac{s^2+16}{s^2+9}\right) - s k_1\left(\frac{k_1^2+16}{k_1^2+9}\right)} = k_1\left(\frac{k_1^2+16}{k_1^2+9}\right)\cdot\frac{k_1^2(k_1^2+16)(s^2+9) - s^2(s^2+16)(k_1^2+9)}{k_1 s(s^2+16)(k_1^2+9) - s k_1(k_1^2+16)(s^2+9)}$$

$$= k_1\left(\frac{k_1^2+16}{k_1^2+9}\right)\frac{-(k_1^2+9)s^4 + \left[k_1^2(k_1^2+16) - 16(k_1^2+9)\right]s^2 + 9k_1^2(k_1^2+16)}{k_1 s\left[(k_1^2+9-k_1^2-16)s^2 + (16k_1^2+16\times9 - 9k_1^2 - 16\times9)\right]}$$

$$= \frac{1}{s}\left(\frac{k_1^2+16}{k_1^2+9}\right)\frac{-(k_1^2+9)s^4 + \left[k_1^4-16\times9\right]s^2 + \left[9k_1^4 + 9\times16k_1^2\right]}{-7s^2+7k_1^2}$$

as Ev $y(s) = 0$ any real positive k allows $(s+k)(s-k) = s^2-k^2$ to cancel

To factor numerator

$$\begin{array}{r|l} & -(k_1^2+9)s^2 - 9[k_1^2+16] \\ \hline s^2-k_1^2 & -(k_1^2+9)s^4 + [k_1^4-16\times9]s^2 + 9k_1^2[k_1^2+16] \\ & -(k_1^2+9)s^4 + [k_1^4+9k_1^2]s^2 \\ \hline & 0 \quad -[9k_1^2+16\times9]s^2 \\ & -[9k_1^2+16\times9]s^2 + 9k_1^2[k_1^2+16] \\ \hline & 0 \qquad\qquad 0 \end{array}$$

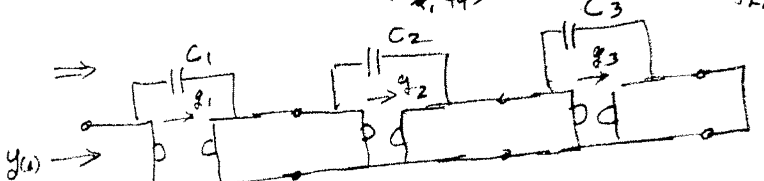
$$\therefore\; y_L(s) = \frac{1}{s}\left(\frac{k_1^2+16}{k_1^2+9}\right)\left(\frac{-(k_1^2+9)}{-7}\right)\frac{s^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{1} = \left(\frac{k_1^2+16}{7}\right)\left[\frac{s^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{s}\right]$$

$$y_{LL}(s) = \frac{k_1^2+16}{7}\cdot\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{k_2}\cdot\frac{k_2\left(\frac{k_1^2+16}{7}\right)\left(\frac{s^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{k_2}\right) - s\left(\frac{k_1^2+16}{7}\right)\left[\frac{s^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{s}\right]}{k_2\left(\frac{k_1^2+16}{7}\right)\left(\frac{s^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{s}\right) - s\left(\frac{k_1^2+16}{7}\right)\left[\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{k_2}\right]}$$

$$= \frac{k_1^2+16}{7}\cdot\left(k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)\right)\cdot s\cdot\frac{\left(k_2^2+9\left(\cancel{\frac{k_1^2+16}{k_1^2+9}}\right)\right) - s^2 - 9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{k_2^2\left(\cancel{s^2}+9\left(\frac{k_1^2+16}{k_1^2+9}\right)\right) - s^2\left(\cancel{k_2^2}+9\left(\frac{k_1^2+16}{k_1^2+9}\right)\right)}$$

$$\therefore\; y_{LL}(s) = \left(\frac{k_1^2+16}{7}\right)\left(\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{9\left(\frac{k_1^2+16}{k_1^2+9}\right)}\right)\cdot s \quad\Rightarrow y_{LLL}(s) = \frac{k}{0} = \infty = \text{short}$$

$$y_{LL}(k_3) = k_3\left(\frac{k_1^2+16}{7}\right)\left(\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{9\left(\frac{k_1^2+16}{k_1^2+9}\right)}\right)$$



$$g_1 = y(k_1) = \frac{k_1(k_1^2+16)}{k_1^2+9}$$

$$C_1 = y(k_1)/k_1 = \left(\frac{k_1^2+16}{k_1^2+9}\right)$$

$$g_2 = y_L(k_2) = \left(\frac{k_1^2+16}{7}\right)\left(\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{k_2}\right)$$

$$C_2 = \left(\frac{k_1^2+16}{7}\right)\left(\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{k_2^2}\right)$$

$$g_3 = y_{LL}(k_3) = \left(\frac{k_1^2+16}{7}\right)\left(\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{9\left(\frac{k_1^2+16}{k_1^2+9}\right)}\right)\cdot k_3$$

$$C_3 = \left(\frac{k_1^2+16}{7}\right)\left(\frac{k_2^2+9\left(\frac{k_1^2+16}{k_1^2+9}\right)}{9\left(\frac{k_1^2+16}{k_1^2+9}\right)}\right)$$

Right off there do not appear to be any advantageous k but k_2 can be chosen so that any two (but, probably not 3) of the C's are equal.

b) If all $k_i = k$ then

$$g_1 = k\left(\frac{k^2+16}{k^2+9}\right), \quad g_2 = \left(\frac{k^2+16}{7}\right)\left(\frac{k^2+9\left(\frac{k^2+16}{k^2+9}\right)}{k}\right), \quad g_3 = k\left(\frac{k^2+16}{7}\right)\left(\frac{k^2+9\left(\frac{k^2+16}{k^2+9}\right)}{9\left(\frac{k^2+16}{k^2+9}\right)}\right)$$

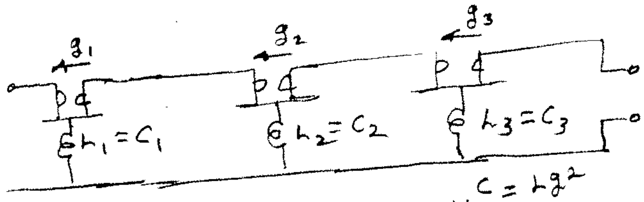
$$C_1 = \left(\frac{k^2+16}{k^2+9}\right), \quad C_2 = \left(\frac{k^2+16}{7}\right)\left(\frac{k^2+9\left(\frac{k^2+16}{k^2+9}\right)}{k}\right), \quad C_3 = \left(\frac{k^2+16}{7}\right)\left(\frac{k^2+9\left(\frac{k^2+16}{k^2+9}\right)}{9\left(\frac{k^2+16}{k^2+9}\right)}\right)$$

showing that they all differ.

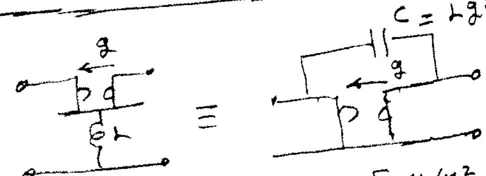
c) Since Z_1 is the dual of Y we can use the dual at each stage

C in parallel → L in series

gyrator $Y = \begin{bmatrix}0 & g\\ -g & 0\end{bmatrix} = Z = \begin{bmatrix}0 & r\\ -r & 0\end{bmatrix} \Rightarrow Y_{dual} = \begin{bmatrix}0 & r\\ -r & 0\end{bmatrix}^{-1} = \begin{bmatrix}0 & -g\\ g & 0\end{bmatrix} \Rightarrow$ turned around gyrator



But



so can replace each section

$$Z = \begin{bmatrix} sL & sL+r\\ sL-r & sL\end{bmatrix} \Rightarrow Y = \begin{bmatrix} sL/r^2 & -\frac{sL}{r^2}-\frac{r}{r^2}\\ -\frac{sL}{r^2}+\frac{r}{r^2} & \end{bmatrix}$$

d) $2\,\mathrm{Ev}\,y(s) = \frac{s+a}{s+b} + \frac{-s+a}{-s+b} = -s^2 - sa + sb + ab - s^2 - sb + sa + ab = 2(-s^2+ab)$

∴ $s_{1,2} = \pm\sqrt{ab}$ are zeros of $\mathrm{Ev}\,y(s)$. $k = +\sqrt{ab}$ is the real positive one.

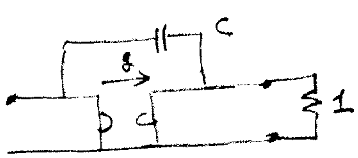
$$y_L(s) = \frac{\sqrt{ab}+a}{\sqrt{ab}+b}\cdot\frac{\sqrt{ab}\left(\frac{\sqrt{ab}+a}{\sqrt{ab}+b}\right) - s\left(\frac{s+a}{s+b}\right)}{\sqrt{ab}\left(\frac{s+a}{s+b}\right) - s\left(\frac{\sqrt{ab}+a}{\sqrt{ab}+b}\right)} = \frac{\sqrt{ab}(\sqrt{ab}+a)(s+b) - (\sqrt{ab}+b)s(s+a)}{\sqrt{ab}(\sqrt{ab}+b)(s+a) - (\sqrt{ab}+a)s(s+b)}\cdot\left(\frac{\sqrt{ab}+a}{\sqrt{ab}+b}\right)$$

$$= \left(\frac{\sqrt{ab}+a}{\sqrt{ab}+b}\right)\frac{-(\sqrt{ab}+b)s^2 + [ab + a\sqrt{ab} - a(\sqrt{ab}+b)]s + \sqrt{ab}\cdot b(\sqrt{ab}+a)}{-(\sqrt{ab}+a)s^2 + [ab+b\sqrt{ab} - b(\sqrt{ab}+a)]s + \sqrt{ab}\cdot a(\sqrt{ab}+b)}$$

$(s-\sqrt{ab})(s+\sqrt{ab}) = s^2 - ab$ should cancel

$$\begin{array}{r|l} & -(\sqrt{ab}+b) \\ s^2-ab & -(\sqrt{ab}+b)s^2 + b(\sqrt{ab}+a)\sqrt{ab} \\ & -(\sqrt{ab}+b)s^2 + ab(\sqrt{ab}+b) \end{array} \qquad \begin{array}{r|l} & -(\sqrt{ab}+a) \\ s^2-ab & -(\sqrt{ab}+a)s^2 + \sqrt{ab}\cdot a(\sqrt{ab}+b) \\ & -(\sqrt{ab}+a)s^2 + ab(\sqrt{ab}+a) \end{array}$$

∴ $y_L(s) = \left(\frac{\sqrt{ab}+a}{\sqrt{ab}+b}\right)\cdot\frac{-(\sqrt{ab}+b)}{-(\sqrt{ab}+a)} = 1$



$$g = y(k) = \frac{\sqrt{ab}+a}{\sqrt{ab}+b}$$

$$C = y(k)/k = \frac{1}{\sqrt{ab}}\cdot\frac{\sqrt{ab}+a}{\sqrt{ab}+b} = \frac{\sqrt{ab}+a}{ab+b\sqrt{ab}} = \frac{1}{b}$$

This shows the Richards function allows synthesis of some non lossless PR $y(s)$. Using complex zeros of the even part also will work using further theory.