Homework 5, page 1

1. a) $T(\alpha) = [c_1 \, c_2]\begin{bmatrix} -a_{11} & \alpha e_{12} \\ 0 & -a_{22}\end{bmatrix}^{-1}\begin{bmatrix} b_1 \\ b_2\end{bmatrix} = [c_1 \, c_2]\begin{bmatrix} -a_{22} & -\alpha e_{12} \\ 0 & -a_{11}\end{bmatrix}\begin{bmatrix} b_1 \\ b_2\end{bmatrix} \times \frac{1}{a_{11}a_{22}}$

$= \begin{bmatrix} -a_{22}c_1 & -\alpha e_{12}c_1 - c_2 a_{11}\end{bmatrix}\begin{bmatrix} b_1 \\ b_2\end{bmatrix} \times \frac{1}{a_{11}a_{22}} = \frac{-\alpha e_{12}c_1 b_2 - a_{22}c_1 b_1 - c_2 a_{11} b_2}{a_{11}a_{22}}$

To obtain $T(\alpha) = \alpha$ choose $-\frac{e_{12}c_1 b_2}{a_{11}a_{22}} = 1$, $b_1 = 0$, $c_2 = 0$

To obtain $T(\alpha) = 1$ choose $b_2 = 0$, $-\frac{c_1 b_1}{a_{11}} = 1$

b) $T(\alpha) = [C_1 \, C_2]\begin{bmatrix} \alpha E_{11} & -A_{12} \\ -A_{21} & 0\end{bmatrix}^{-1}\begin{bmatrix} B_1 \\ B_2\end{bmatrix} = [C_1 \, C_2]\begin{bmatrix} 0 & +A_{12} \\ +A_{21} & \alpha E_{11}\end{bmatrix}\begin{bmatrix} B_1 \\ B_2\end{bmatrix} \times \frac{1}{A_{21}A_{12}}$

$= \begin{bmatrix} C_2 A_{21} & C_1 A_{12} + C_2 \alpha E_{11}\end{bmatrix}\begin{bmatrix} B_1 \\ B_2\end{bmatrix} \times \frac{1}{A_{12}A_{21}} = \frac{\alpha C_2 B_2 E_{11} + C_2 A_{21} B_1 + C_1 A_{12} B_2}{A_{21}A_{12}}$

which by choices: $-\frac{e_{12}c_1 b_2}{a_{11}a_{22}} = \frac{C_2 B_2 E_{11}}{A_{21}A_{12}}$ and

(note: choose $B_1 = C_1 = 0$, $C_2 = B_2 = E_{11} = A_{21} = A_{12} = 1$)

$$\frac{-a_{22}c_1 b_1 - c_2 a_{11} b_2}{a_{11}a_{22}} = \frac{C_2 A_{21} B_1 + C_1 A_{12} B_2}{A_{21}A_{12}}$$

will give the ~~same~~ $T(\alpha)$.

c) $\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22}\end{bmatrix}\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22}\end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22}\end{bmatrix} = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0\end{bmatrix}$ and

$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22}\end{bmatrix}\begin{bmatrix} 0 & e_{12} \\ 0 & 0\end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22}\end{bmatrix} = \begin{bmatrix} E_{11} & 0 \\ 0 & 0\end{bmatrix} \Rightarrow \begin{bmatrix} 0 & p_{11}e_{12} \\ 0 & p_{21}e_{12}\end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22}\end{bmatrix}$

$= \begin{bmatrix} p_{11}e_{12}q_{21} & p_{11}e_{12}q_{22} \\ p_{21}e_{12}q_{21} & p_{21}e_{12}q_{22}\end{bmatrix} \Rightarrow p_{21} = 0$, $q_{22} = 0$, $E_{11} = p_{11}e_{12}q_{21}$

$\Rightarrow \begin{bmatrix} p_{11} & p_{12} \\ 0 & p_{22}\end{bmatrix}\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22}\end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & 0\end{bmatrix} = \begin{bmatrix} p_{11}a_{11} & p_{12}a_{22} \\ 0 & p_{22}a_{22}\end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & 0\end{bmatrix}$

$= \begin{bmatrix} p_{11}a_{11}q_{11} + p_{12}a_{22}q_{21} & p_{11}a_{11}q_{12} \\ p_{22}a_{22}q_{21} & 0\end{bmatrix} \Rightarrow p_{12}a_{22}q_{21} = -p_{11}a_{11}q_{11}$

$\therefore$ choose $p_{11} = \frac{E_{11}}{e_{12}q_{21}}$, $p_{11}a_{11}q_{12} = A_{12} \Rightarrow q_{12} = \frac{A_{12}}{a_{11}} \cdot \frac{1}{p_{11}} = \frac{A_{12}}{a_{11}} \cdot \frac{e_{12}}{E_{11}} \cdot q_{21}$

& $p_{22}a_{22}q_{21} = A_{21} \Rightarrow p_{22} = \frac{A_{21}}{a_{22}} \cdot \frac{1}{q_{21}}$ & $p_{12} = \frac{-p_{11}a_{11}q_{11}}{a_{22}q_{21}} = -\frac{E_{11}}{e_{12}} \cdot \frac{1}{q_{21}^2} \cdot \frac{q_{11}}{a_{22}}$

$\therefore P = \begin{bmatrix} \frac{E_{11}}{e_{12}} \cdot \frac{1}{q_{21}} & -\frac{E_{11}}{e_{12}} \cdot \frac{1}{q_{21}^2} \frac{q_{11}}{a_{22}} \\ 0 & \frac{A_{21}}{a_{12}} \cdot \frac{1}{q_{21}}\end{bmatrix}$, $Q = \begin{bmatrix} q_{11} & \frac{A_{12}}{a_{11}} \frac{e_{12}}{E_{11}} q_{21} \\ q_{21} & 0\end{bmatrix}$ with $q_{21} \neq 0$ & q_{11} free to choose.

10/31/09
RWN, Y.Z

Homework 5, #1 cont.

d) $T(s) = s = v/i \equiv$ an inductor

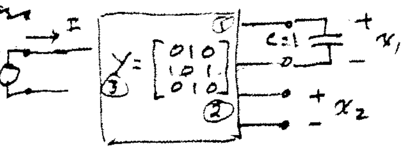
From the above, use the E_{11} form

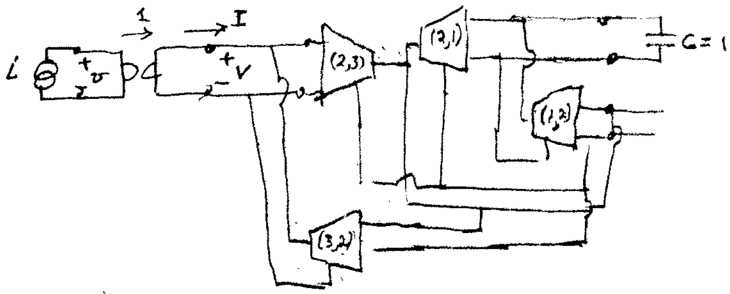
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} i$$

$$v = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

To make this with OTA-C, as $u = i$, $y = v$, consider as Y and convert port I to v & V to i via gyrators

$$y = \begin{bmatrix} E s\, x \\ I \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ V = u \end{bmatrix} \Rightarrow$$



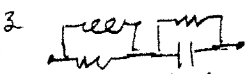


all gains $g_m = 1$

#2. a) $f(s)$ is PR & lossless, $f(s) = -f(-s)$, poles & zeros simple on $j\omega$ axis alternating

∴ not BR as poles on $j\omega$ axis

b) $f(s)$ is PR as $\frac{as}{cs+d} + \frac{b}{cs+d} = \frac{1}{\frac{c}{a} + \frac{1}{\frac{a}{d}s}} + \frac{1}{\frac{cs}{b} + \frac{d}{b}} \Rightarrow$ (as Z)



with positive R, L, C's

as all coefficients are > 0 it is not lossless

To check for BR $1 - \frac{b^2 + a^2\omega^2}{d^2 + c^2\omega^2} = \frac{d^2 - b^2 + (c^2 - a^2)\omega^2}{d^2 + c^2\omega^2}$

which is > 0 if $d^2 > b^2$ & $c^2 > a^2 \Rightarrow$ BR otherwise not

c) as there is a zero in $\sigma > 0$ which does not cancel this can not be PR.

It is BR since it is analytic in $\sigma > 0$ and

$$|f(j\omega)|^2 = 1 = \frac{\sqrt{2^2+\omega^2}}{\sqrt{2^2+\omega^2}} \cdot \frac{\sqrt{(4-\omega^2)^2 + (4\omega)^2}}{\sqrt{(4-\omega^2)^2 + (4\omega)^2}}$$

as $f(-s) = \frac{1}{f(s)}$ this is a lossless BR scattering matrix