Homework 3, #1

1. $Z_{cc}(s) = sL\begin{bmatrix} 1 & -k \\ -k & k^2 \end{bmatrix}$, $\det Z_{cc}(s) = (sL)^2(1\cdot k^2 - (-k)^2) \equiv 0 \Rightarrow$ no $Y = Z^{-1}$

$$Z_{cap}(s) = \frac{1}{sC}\begin{bmatrix} 1 & +1 \\ +1 & 1 \end{bmatrix}$$

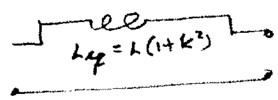
$$Z_{2\text{-port}}(s) = Z_{cc}(s) + Z_{cap}(s) = \begin{bmatrix} sL + \frac{1}{sC} & -sLk + \frac{1}{sC} \\ -sLk + \frac{1}{sC} & sLk^2 + \frac{1}{sC} \end{bmatrix}$$

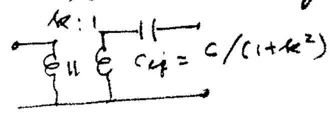
$$\det Z_{2\text{port}} = \left(sL + \tfrac{1}{sC}\right)\left(sLk^2 + \tfrac{1}{sC}\right) - \left(-sLk + \tfrac{1}{sC}\right)^2$$
$$= \cancel{s^2L^2k^2} + \frac{L}{C} + \frac{L}{C}k^2 + \cancel{\frac{1}{s^2C^2}} - \left[\cancel{s^2L^2k^2} - 2\frac{Lk}{C} + \cancel{\frac{1}{s^2C^2}}\right]$$
$$= \frac{L}{C}\left[1 + 2k + k^2\right] = \frac{L}{C}(1+k)^2$$

$$\therefore Y_{2\text{port}} = Z^{-1}_{2\text{-port}} = \frac{1}{\frac{L}{C}(1+k^2)}\begin{bmatrix} sLk^2 + \frac{1}{sC} & sLk - \frac{1}{sC} \\ sLk - \frac{1}{sC} & sL + \frac{1}{sC} \end{bmatrix}$$

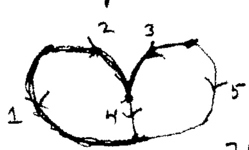
$$= \underbrace{\frac{sC}{(1+k^2)}\begin{bmatrix} k^2 & k \\ k & 1 \end{bmatrix}}_{Y_{cap}(s)} + \underbrace{\frac{1}{sL(1+k^2)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{Y_{ind}(s)}$$

$= Y_{cap}(s) + Y_{ind}(s)$ indicating the parallel connection of two 2-ports

$Y_{ind} \Rightarrow$ 

, $Y_{cap} =$ 

here the capacitor is a mutually coupled one, coupled through an ideal transformer

b) 

$i_b = [i_1, i_2, i_3, i_4, i_5]^T + [-I_{in}, 0, 0, 0, 0]^T = i + J$, $i_\ell = [i_4, i_5]_b^T$, $v_t = [v_1, v_2, v_3]_b^T$

$v_b = [v_1, v_2, v_3, v_4, v_5]^T = v$

$$Av = Bi \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{$[Y_{2\text{-port}}]$}} & 0 & 0 \\ 0 & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} i \quad \begin{matrix} \Rightarrow i_4 = 0 \\ \Rightarrow v_4 = 0 \end{matrix}$$

cutsets: $\underline{Q}_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} i_b$;

tie sets: $\underline{Q}_2 = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 1 \end{bmatrix} v_b$

$$v_b = v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{bmatrix} v_t, \quad i_b = i + J = \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} i_\ell \Rightarrow$$

as $i_4 = 0$ then $i_1 = -I_{in} = -i_{\ell_2} = -i_5$

$\Rightarrow i_5 = I_{in} = -i_3 = i_2$

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Y_{2\text{-port}} \end{bmatrix}\begin{bmatrix} v_2 \\ v_3 \end{bmatrix} \quad (\text{from } Av = Bi)$$

$\hookrightarrow v_4 = 0 \Rightarrow v_1 = v_2$

$\hookrightarrow v_3 = v_5$

$$\Rightarrow \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = Z_{2\text{port}}\begin{bmatrix} I_{in} \\ -I_{in} \end{bmatrix}$$

$$\Rightarrow v_5 = v_3 = z_{21}I_{in} - z_{22}I_{in} = \left[-sLk + \tfrac{1}{sC} - \left(sLk^2 + \tfrac{1}{sC}\right)\right]I_{in}$$

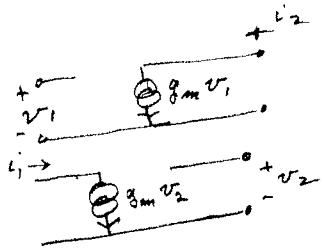
$\underline{v_0 = -sL(k + k^2)\,I_{in}}$ (mutual + output) inductance

Homework 3, #2

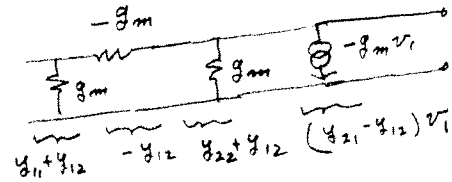
#2.

a) $Y=\begin{bmatrix}0&0\\g_m&0\end{bmatrix} \Rightarrow$

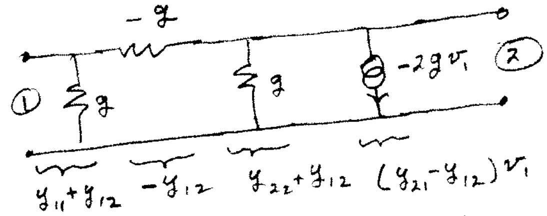
$Y=\begin{bmatrix}0&g_m\\0&0\end{bmatrix} \Rightarrow$



$=$



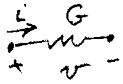
b) $Y=\begin{bmatrix}0&g\\-g&0\end{bmatrix}=\begin{bmatrix}0&g\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\-g&0\end{bmatrix} \Rightarrow$



power $\quad gv_1^2 \quad -g(v_1-v_2)^2 \quad gv_2^2 \quad (-2gv_1)\cdot v_2$

$\Sigma = \cancel{gv_1^2} - g(\cancel{v_1^2} - 2v_1v_2 + \cancel{v_2^2}) + \cancel{gv_2^2} - 2gv_1v_2$

$= 0$ = sum of powers in each component (some are <0)

c) 

$\Rightarrow$ power $= v \cdot i =$
$= v \cdot Gv$
$= Gv^2$

$$V_{port}^T I_{port} = V^T\begin{bmatrix}0&g\\-g&0\end{bmatrix}V$$

$$=[v_1 \;\; v_2]\begin{bmatrix}0&g\\-g&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix} = [-gv_2,\; gv_1]\begin{bmatrix}v_1\\v_2\end{bmatrix} = -gv_2v_1 + gv_1v_2$$

$$= 0$$

∴ the power into the ports = sum of powers in components

$= 0 \Rightarrow$ instantaneously lossless