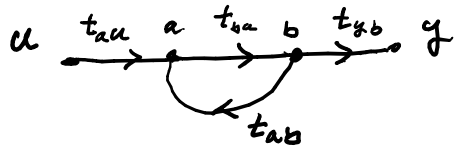
EE 303 H
10/20/09

Feedback



$$x_b = t_{ba} x_a \quad 1)$$

$$x_a = t_{au} \cdot u + t_{ab} x_b \quad 2)$$

$$y = t_{yb} \cdot x_b \quad 3)$$

2) × t_{ba} & 1)

$$t_{ba} x_a = x_b = t_{ba} \cdot t_{au} \cdot u + t_{ba} \cdot t_{ab} x_b \quad \text{solve for } x_b$$

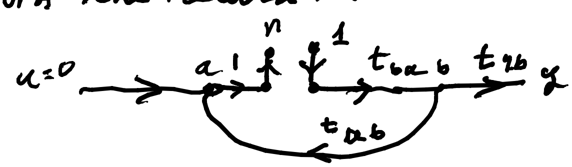
$$x_b = (1 - t_{ba} t_{ab})^{-1} \cdot t_{ba} t_{au} \cdot u$$

3)

$$y = t_{yb} (1 - t_{ba} t_{ab})^{-1} t_{ba} t_{au} \cdot u$$

$$T = t_{yb} (1 - t_{ba} t_{ab})^{-1} t_{ba} t_{au}$$

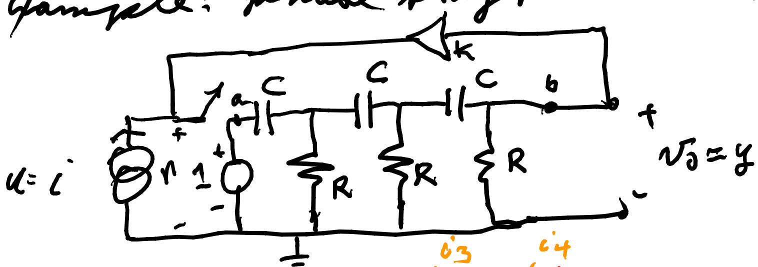
For the return ratio & return difference:

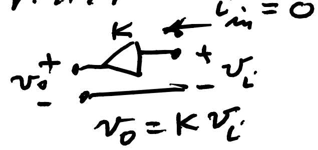


$\Rightarrow r = t_{ab} \cdot t_{ba}$ = return ratio

$d = 1 - r$ = return difference
$= 1 - t_{ab} t_{ba}$ ⇒ in denominator of T
its zeros give the poles of T

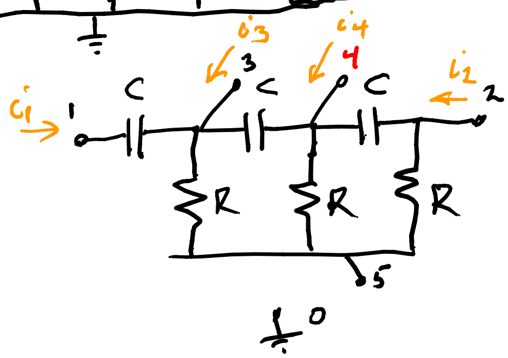
Example: phase shift oscillator, p. 1174





$i_{in} = 0$

$v_o = K v_i$



$G = 1/R$

2-port $Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$; port 1 $= a/\perp$; port 2 $= b/\perp$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad v_1 = v_{in}, \quad v_2 = v_0$$

derive $\frac{v_0}{v_{in}} =$

$i_2 = 0$ = amplifier current input

$$0 = y_{21} v_1 + y_{22} v_2 \Rightarrow \frac{v_2}{v_1} = -y_{21}/y_{22}$$

one way: find indefinite $\underset{(5\times5)}{Y}$, move $\perp$ to node 5

For Y_{indef} = find the currents into the 5 nodes when put voltage sources from nodes to $\perp$

$$= \begin{bmatrix} ac & 0 & -ac & 0 & 0 \\ 0 & ac+G & 0 & -ac & -G \\ -ac & 0 & 2ac+G & -ac & -G \\ 0 & -ac & -ac & 2ac+G & -G \\ 0 & -G & -G & -G & 3G \end{bmatrix}$$

rows $\Sigma = 0$
colum $\Sigma = 0$
quite singular

move ground to node 5 $\rightarrow v_5 = 0$ & ignore i_5 as it sums all other currents

scratches out the rows & columns associated with the ground node:

$$Y_{def} = \left[\begin{array}{cc|cc} ac & 0 & -ac & 0 \\ 0 & ac+G & 0 & -ac \\ \hline -ac & 0 & 2ac+G & -ac \\ 0 & -ac & -ac & 2ac+G \end{array}\right] = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right]$$

next for $i_3 = i_4 = 0$

all Y_{ij} are 2x2

$$\begin{bmatrix} i_1 \\ i_2 \\ \hline 0 \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = Y_{21}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{22}\begin{bmatrix} v_3 \\ v_4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = -Y_{22}^{-1} Y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$ if Y_{22}^{-1} exists

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{11}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + Y_{12}\left(-Y_{22}^{-1} Y_{21}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix};\quad \underline{Y_{2port} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}}$$

need Y_{22}^{-1}: $\det\begin{bmatrix} 2ac+G & -ac \\ -ac & 2ac+G \end{bmatrix} = 4a^2c^2 + 4acG + G^2 - a^2c^2$

$= 3a^2c^2 + 4acG + G^2$

y_{21} & y_{22}

$$Y_{2port} = \begin{bmatrix} ac & 0 \\ 0 & ac+G \end{bmatrix} - \frac{\begin{bmatrix} -ac & 0 \\ 0 & -ac \end{bmatrix}\begin{bmatrix} 2ac+G & ac \\ ac & 2ac+G \end{bmatrix}\begin{bmatrix} -ac & 0 \\ 0 & -ac \end{bmatrix}}{3a^2c^2 + 4acG + G^2}$$

$$\text{return ratio} = -\frac{y_{21}}{y_{22}} = \left.\frac{v_2}{v_1}\right|_{i_2=0} = -\frac{\left[-ac^3\right] / \cancel{det}}{\dfrac{(ac+G)\left[3a^2c^2 + 4acG + G^2\right] - (a^2c^2)(2ac+G)}{\cancel{det}}} = r/k$$

When we close the loop desire $d = 1 - r$

desire zeros of d to get poles of transfer function

$$d = 1 - \frac{\left[+ac^3K\right]}{(ac+G)\left[3a^2c^2 + 4acG + G^2\right] - a^2c^2(2ac+G)} = \frac{aa^3 + \cdots}{6a^3 + \cdots}$$

desire the numerator to have a factor $s^2+\omega_0^2$
(without the feedback all zeros are on the negative real axis so the RC ladder will not oscillate)

Problem: To control the amplitude? How?

Look at the Van der Pol oscillator

$$\ddot{x} + \epsilon(x^2-1)\dot{x} + \omega_0^2 x = 0$$

this one is a structurally stable oscillator

1) Let $\dfrac{dy}{dt} = \omega_0^2 x$

2) $$\frac{d^2x}{dt^2} + \epsilon(x^2-1)\frac{dx}{dt} = -\omega_0^2 x = -\frac{dy}{dt}$$

integrate 2)

3) $$\frac{dx}{dt} + \epsilon\left(\frac{x^3}{3} - x\right) = -y$$

$$\left\{\begin{aligned} \frac{dy}{dt} &= \omega_0^2 x \\ \frac{dx}{dt} &= -y + \epsilon\left(x - \frac{x^3}{3}\right) \end{aligned}\right\} \text{ state equations}$$