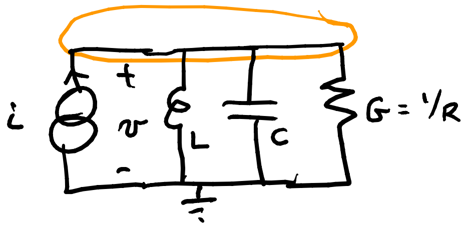
oscillators - p. 1179

EE303H
10/15/09 ⇒ 10/23/09 correction



$$i = \frac{1}{Ls}v + Csv + Gv$$

$$Lsi = (1 + CLs^2 + LGs)v$$

$$= CL\left(s^2 + \frac{G}{C}s + \frac{1}{LC}\right)v$$

$$\frac{1}{C}si = \left(s^2 + \frac{G}{C}s + \frac{1}{LC}\right)v$$

$-\infty < t < \infty$

Let $i(t) = Ie^{s_0 t}$

assume $v(t) = Ve^{s_0 t}$

$$v = \frac{\frac{1}{C}s}{s^2 + \frac{G}{C}s + \frac{1}{LC}} \cdot i = T(s) \cdot i$$

$$\frac{s_0}{C} I \cancel{e^{s_0 t}} = \left(s_0^2 + \frac{G}{C}s_0 + \frac{1}{LC}\right) V \cancel{e^{s_0 t}}$$

Case of $I = 0 \Rightarrow \quad 0 = \left(s_0^2 + \frac{G}{C}s_0 + \frac{1}{LC}\right)V$ — can have $V \neq 0$ if s_0 satisfies

$$P(s_0) = s_0^2 + \frac{G}{C}s_0 + \frac{1}{LC} = 0$$

zeros of $P(s_0)$ are open circuit $(I=0)$ natural frequency.

$$P(s) = s^2 + \frac{\omega_0}{Q}s + \omega_0^2, \quad \omega_0^2 = \frac{1}{LC}, \quad \omega_0 = +\sqrt{\frac{1}{LC}}$$

$$\frac{\omega_0}{Q} = \frac{G}{C} \Rightarrow Q = \frac{\omega_0 C}{G} = \frac{1}{G}\sqrt{\frac{C}{L}}$$

Q is a measure of damping
(high Q has little damping)

$$s_0 = -\frac{G}{2C} \pm \frac{1}{2}\sqrt{\left(\frac{G}{C}\right)^2 - \frac{4}{LC}} = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2}$$

if $Q = \infty$ then $s_{0_\pm} = \pm j\omega_0$; $e^{s_0 t} = e^{j\omega_0 t}$ & $e^{-j\omega_0 t}$

$= \cos\omega_0 t + j\sin(\omega_0 t)$

if $Q \neq \infty$ have damping meaning

when $Q = \infty$ then the circuit oscillates giving responses that are cosines & sines

any practical circuit has R present so its "oscillations" will die.

ideal oscillator has $P(s) = s^2 + \omega_0^2$

$$si = (s^2 + \omega_0^2)v = \frac{d^2v}{dt^2} + \omega_0^2 v = ($$

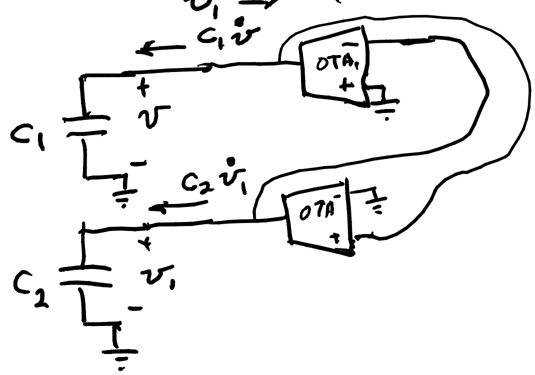
if $i = 0 \Rightarrow v \neq 0 \Rightarrow (s^2 + \omega_0^2)v = 0$

$\dot v = v_1$

$\dot v_1 = \ddot v = -\omega_0^2 v$ $\Big\}$ $\dot v = v_1$, $\dot v_1 = -\omega_0^2 v$

take v_1 = current from $v_1 \Rightarrow$ OTA

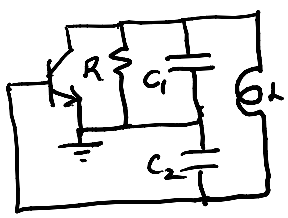
$\dot v_1 \Rightarrow$ " " in a capacitor

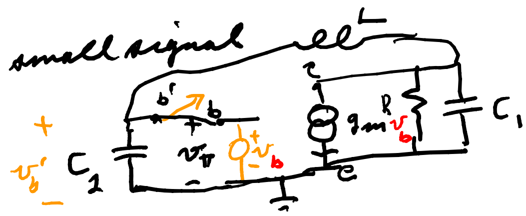


gain of OTA$_1$ = $1/C_1$

gain of OTA$_2$ = $\frac{\omega_0^2}{C_2}$

Let's look at the Colpitts oscillator, p. 1180



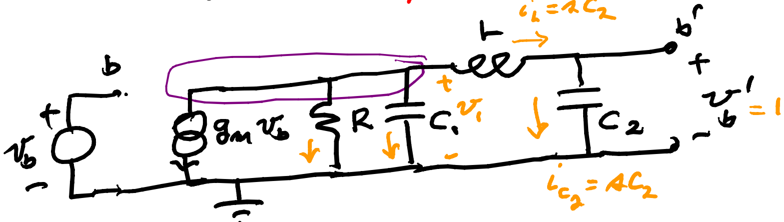


(bias not shown here but on p. 1182)

redraw to get the return ratio v_b'/v_b

desire = 1 for an oscillator

Find $v_b' = T(s)\, v_b$



If $v_b' = 1$ then find v_b & force $v_b = 1$ to oscillate

$v_1 = \underset{v_b'}{1} + Ls(sC_2) = LC_2 s^2 + 1$

$i_{C_1} = sC_1 v_1 = sC_1(LC_2 s^2 + 1)$

$i_R = G v_1 = G(LC_2 s^2 + 1)$

inductor "

by KCL: $g_m v_b + GLC_2 s^2 + G + s^3 C_1 L C_2 + sC_1 = -sC_2$

here $v_b' = 1$; $v_b = \left(-sC_2 - GLC_2 s^2 - G - s^3 C_1 L C_2 - sC_1\right) \cdot \frac{1}{g_m}$

$\therefore \quad T(s) = \frac{v_b' = 1}{v_b} = \frac{-g_m}{s^3 L C_1 C_2 + s^2 (G L C_2) + s(C_2 + C_1) + G}$

desire $v_b = 1 \Rightarrow s^3 L C_1 C_2 + s^2 (G L C_2) + s(C_1 + C_2) + G + g_m = 0$

we desire two known imaginary roots! $s = j\omega_0$

set $s = j\omega_0$ & force coefficients to give this

$$-j\omega_0^3 L C_1 C_2 - \omega_0^2 G L C_2 + j\omega_0 (C_1 + C_2) + G + g_m = 0$$

requires real & imaginary parts both $= 0$

$$j\omega_0 \{ C_1 + C_2 - \omega_0^2 L C_1 C_2 \} = 0 \qquad -\omega_0^2 G L C_2 + G + g_m = 0$$

$$\Rightarrow \omega_0 = \sqrt{\underbrace{\frac{C_1 + C_2}{L C_1 C_2}}_{\text{a series } C = \frac{C_1 C_2}{C_1 + C_2}}} \qquad \Rightarrow \quad -\frac{(C_1 + C_2)}{L C_1 C_2} \cdot G L C_2 + G + g_m = 0$$

in parallel with inductor L

$$-\frac{(\cancel{C_1} + C_2) G}{C_1} = -\cancel{G} - g_m$$

$$g_m = G\left(\cancel{1} + \left(\cancel{1} + \frac{C_2}{C_1}\right)\right)$$

$$= G\left(\cancel{2} + \frac{C_2}{C_1}\right) = \frac{C_2}{C_1} G$$

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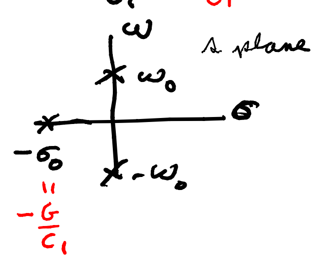
allows factorization of the

$$P(s) = (s^2 + \omega_0^2)(s + \sigma_0)$$

$$= s^3 + \frac{G}{C_1} s^2 + \left(\frac{C_2 + C_1}{L C_1 C_2}\right) s + \frac{G + g_m}{L C_1 C_2}$$

$\Rightarrow \sigma_0 = \frac{G}{C_1}$



have created a Colpitts oscillator