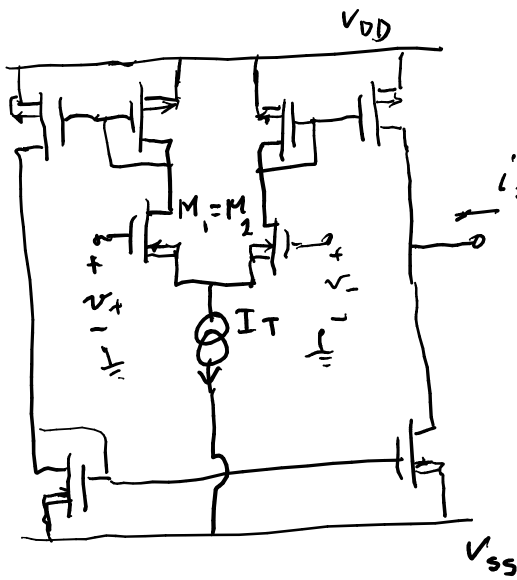
EE 303H
10/08/09



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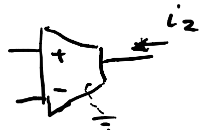
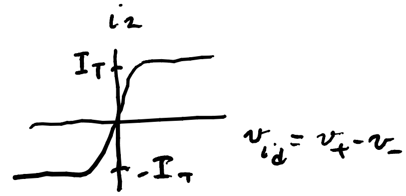
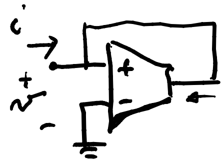
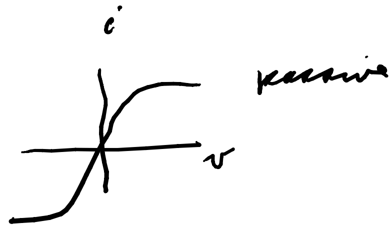


an OTA



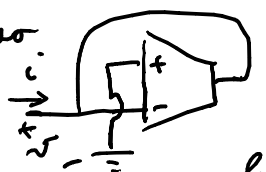
$$G_m = \sqrt{2\frac{KP}{2}\frac{W}{L} I_T} = \text{slope at } v_{id} = 0$$

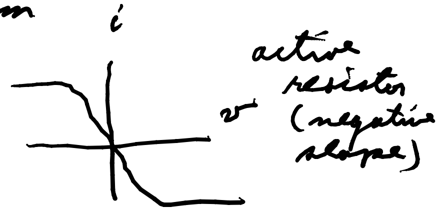




this is like a resistor of value $R = \frac{1}{G_m}$ at the origin

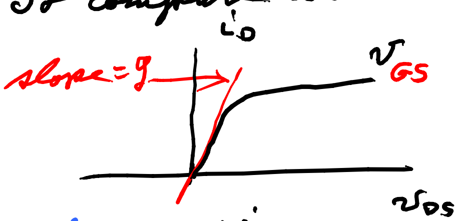
But also





here $R = -\frac{1}{G_m}$

To compare with R from an NMOS

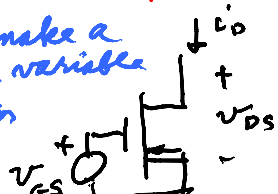


near the origin

$$i_D = \frac{KP}{2}\frac{W}{L}\left(2(V_{GS} - V_{TO_n})v_{DS} - v_{DS}^2\right)$$

for small v_{DS}

$$i_D \sim \left[\frac{KP}{2}\frac{W}{L} \cdot 2(V_{GS} - V_{TO_n})\right] v_{DS}$$



This make a voltage variable resistor

$$g = \frac{i_D}{v_{DS}} = \frac{\partial i_D}{\partial v_{DS}} = \frac{KP}{2} \cdot \frac{W}{L} \cdot 2(V_{GS} - V_{TO_n})$$

Ex: $KP = 9\times10^{-5}$, $I_T = 10^{-3}$, $\frac{W}{L} = 4$, $V_{GS} = 5$, $V_{TO_n} = 1$

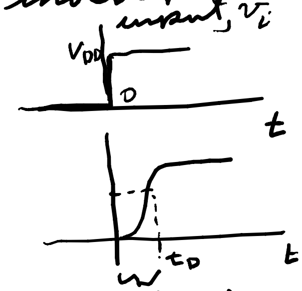
$$G_m = \sqrt{KP \cdot \frac{W}{L} I_T} = \sqrt{9\times10^{-5} \cdot 4\times10^{-3}} = 6\times10^{-4}\,\mho; \quad R = \frac{10}{6}\times10^{3} = 1.33\,K\Omega$$

for OTA

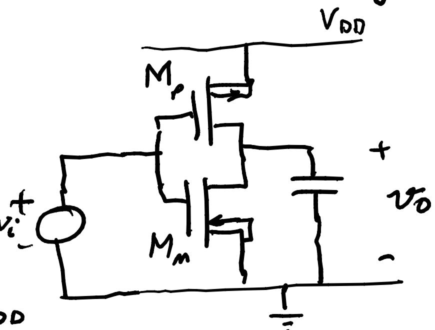
$$g_{NMOS} = 9\times10^{-5}\times4(5-1) = 36\times4\times10^{-5} = 144\times10^{-5} = 1.44\times10^{-3}\,\mho$$

$$R = \frac{1}{1.44}\times10^{3} \approx 0.8\,K\Omega$$

Back to the inverter



w delay time tells how fast can switch

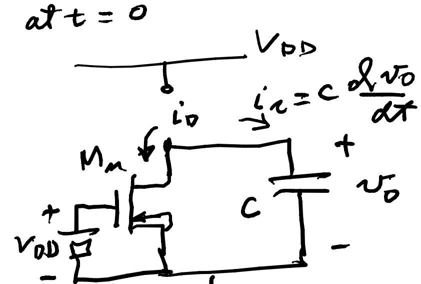


steps v_i from 0 to V_{DD} $= v_i$

at $t=0$ assume $v_o = V_{DD}$

switch at $t=0$

at $t=0$



desire to find time to switch to $v_o = \frac{V_{DD}}{2}$

@ $t=0$ $V_{GS}-V_{TO_n} = V_{DD}-V_{TO_n} < V_{DD} = v_{DS}$ $\Downarrow$

if M_n is enhancement

M_n is in the saturation state

$$\therefore \quad i_D = \frac{KP}{2}\frac{W}{L}(V_{GS}-V_{TO_n})^2; \quad \text{let } K = \frac{KP}{2}\frac{W}{L}$$

$$i_c = C\frac{dv_o}{dt_t} = -i_D = -K(V_{DD}-V_{TO_n})^2$$

Solve:

$$\int_0^t C\frac{dv_o}{d\tau}\,d\tau = C\int_{v_o(0)}^{v_o(t)} dv_o = C\,v_o(t) - C\,v_o(0)$$

$$= -K(V_{DD}-V_{TO_n})^2\int_0^t d\tau = -K(V_{DD}-V_{TO_n})^2\,t$$

eventually $v_{DS} = v_O$ becomes $= v_{GS} - V_{TO_n} = V_{DD} - V_{TO_n}$

$$C\, v_O(t) = C V_{DD} - K(V_{DD} - V_{TO_n})^2 t$$

desire t_0 @ which $v_O(t_0) = v_{DS}(0) = V_{DD} - V_{TO_n}$

$$v_O(t_0) = \cancel{V_{DD}} - V_{TO_n} = \cancel{V_{DD}} - \frac{K}{C}(V_{DD} - V_{TO_n})^2 t_0$$

$$t_0 = \frac{V_{TO_n}}{(V_{DD} - V_{TO_n})^2} \cdot \frac{C}{K}$$

@ this time we switch to the ohmic region

$$C\frac{dv_0}{dt} = -i_D = -K\left(2(V_{GS} - V_{TO_n})v_0 - v_0^2\right)$$

this is a Riccati equation

$$\frac{dv_0}{dt} = -\frac{K}{C}\left(a v_0 - v_0^2\right) \qquad a = 2(V_{GS} - V_{TO_n}) = 2(V_{DD} - V_{TO_n})$$

$$dv_0 = -\frac{K}{C}(a v_0 - v_0^2)\,dt \Rightarrow -\frac{C}{K}\frac{dv_0}{(a v_0 - v_0^2)} = dt$$

$$-\frac{C}{K}\int_{v_0(t_0)}^{v_0(t)} \frac{dv_0}{v_0(a - v_0)} = \int_{t_0}^{t} d\tau = t - t_0$$

make a partial fraction expansion of $\frac{1}{x(a-x)}$, $x = v_0$

$$\frac{1}{x(a-x)} = \frac{k_1}{x} + \frac{k_2}{x-a}\ ; \quad k_1 + \frac{k_2 x}{x-a}\bigg|_{x=0} = \frac{x}{x(a-x)}\bigg|_{x=0} = \frac{1}{a-x}\bigg|_{x=0}$$

$$= \frac{1/a}{x} + \frac{-1/a}{x-a} \qquad = k_1 = 1/a$$

check by cross multiplying

$$k_2 = \frac{x-a}{x(a-x)}\bigg|_{x=a} - \frac{k_1(x-a)}{x}\bigg|_{x=a} = \frac{-1}{a} + 0$$

$$\frac{\frac{1}{a}(x-a) + (-1/a)x}{x(x-a)} = \frac{-1}{x(x-a)}$$

$$-\frac{C}{K}\left[\int_{v(t_0)}^{v(t)} \frac{\frac{1}{a}dv_0}{v_0} + \int_{v(t_0)}^{v(t)} \frac{-1/a\, d(v_0 - a)}{v_0 - a}\right] = -\frac{C}{K}\left[\frac{1}{a}\ln v_0\bigg|_{v_0(t_0)}^{v_0(t)} - \frac{1}{a}\ln(v_0 - a)\bigg|_{v_0 = v(t_0)}^{v(t)}\right]$$

$$= -\frac{C}{aK}\left[\ln\left(\frac{v_0}{v_0 - a}\right)\bigg|_{v_0(t_0)}^{v_0(t)}\right] = t - t_0$$

find t for which $v_0(t) = \frac{V_{DD}}{2}$ $\Rightarrow$ $t_D = t_0 + \left(-\frac{C}{aK}\left[\ln\left(\frac{V_{DD}/2}{\frac{V_{DD}}{2} - a}\right) - \ln\left(\frac{v_0(t_0)}{v_0(t_0) - a}\right)\right]\right)$

$$\frac{a}{2} = V_{DD} - V_{TO_m}\ , \quad t_o = \dot{t}_o = \frac{V_{TOm}}{(V_{DD} - V_{TO_m})^2} \cdot \frac{C}{K}\ , \quad v_o(t_o) = \frac{a}{2}$$

$$\frac{V_{DD}}{2} - a = \frac{V_{DD}}{2} - 2(V_{DD} - V_{TO_m}) = -\frac{3}{2}V_{DD} + 2V_{TO_m}$$

$$t_D = t_o - \frac{C}{aK}\left[\ln\left(\left(\frac{V_{DD}/2}{\frac{V_{DD}}{2} - a}\right) \Big/ \left(\frac{a/2}{\frac{a}{2} - a}\right) \right) \right] = t_o - \frac{C}{aK} \ln\left(\frac{V_{DD}/2}{a - \frac{V_{DD}}{2}} \right)$$

$$= \frac{C}{K}\left\{ \frac{V_{TOm}}{(V_{DD} - V_{TO_m})^2} + \ln\left(\frac{\frac{3}{2}V_{DD} - 2V_{TO_m}}{V_{DD}/2} \right) \right\}$$

the delay to move a pulse through an inverter