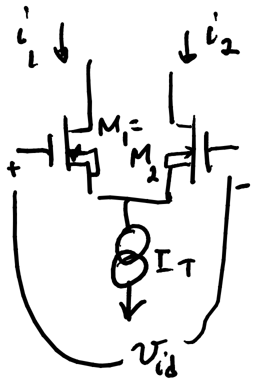
EE303H
10/06/09



$i_o = i_2 - i_1$

$I_T = i_1 + i_2 \Rightarrow i_2 = I_T - i_1$

$i_1 = K_1(v_{GS_1} - V_{TO})^2, \; i_2 = K_1(v_{GS_2} - V_{TO})^2$

$K_1 = \frac{KP}{2}\frac{W}{L}$

$v_{GS_1} = V_{TO} \pm \sqrt{i_1/K_1}, \quad v_{GS_2} = V_{TO} \pm \sqrt{i_2/K_1}$ as $v_{GS} > V_{TO}$ need + signs

$v_{id} = v_{GS_1} - v_{GS_2} = \sqrt{i_1/K_1} - \sqrt{i_2/K_1}$

$K_1 v_{id}^2 = i_1 + i_2 - 2\sqrt{i_1 i_2} = I_T - 2\sqrt{i_1 i_2} \Rightarrow \sqrt{i_1 i_2} = \frac{I_T - K_1 v_{id}^2}{2}$

square $i_1 i_2 = \frac{(I_T - K_1 v_{id}^2)^2}{4} \Rightarrow i_1^2 - I_T i_1 + \frac{1}{4}(I_T - K_1 v_{id}^2)^2 = 0$

$(i_2 = I_T - i_1)$

$i_1(I_T - i_1)$

solve the quadratic for i_1

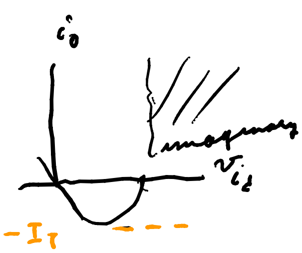
$$i_1 = \frac{I_T}{2} \pm \frac{1}{2}\sqrt{(-I_T)^2 - 4 \times \frac{1}{4}(I_T - K_1 v_{id}^2)^2}$$

$$= \frac{I_T}{2}\left[1 \pm \sqrt{1 - \underbrace{\left(1 - \frac{K_1}{I_T} \cdot v_{id}^2\right)^2}_{1 - 2\frac{K_1}{I_T}v_{id}^2 + \left(\frac{K_1}{I_T}\right)^2 v_{id}^4}}\right]$$

$$= \frac{I_T}{2}\left[1 \pm \sqrt{v_{id}^2 \times \frac{K_1}{I_T}\left(2 - \frac{K_1}{I_T}v_{id}^2\right)}\right]$$

$$= \frac{I_T}{2}\left[1 \pm v_{id}\boxed{\frac{K_1}{I_T}}\sqrt{2 - \frac{K_1}{I_T}v_{id}^2}\right]$$ choose + sign

$$i_2 = I_T - i_1 = \frac{I_T}{2}\left[1 - \sqrt{\frac{K_1}{I_T}}v_{id}\sqrt{2 - \left(\sqrt{\frac{K_1}{I_T}} \cdot v_{id}\right)^2}\right]$$



for an OTA $\quad i_o = i_2 - i_1$

$$i_o = i_2 - i_1 = -I_T \cdot \sqrt{\frac{K_1}{I_T}} \cdot v_{id}\sqrt{2 - \left(\sqrt{\frac{K_1}{I_T}} \cdot v_{id}\right)^2}$$

~~Max~~ Min of i_o vs v_{id}; $\quad x = \sqrt{\frac{K_1}{I_T}}v_{id}$

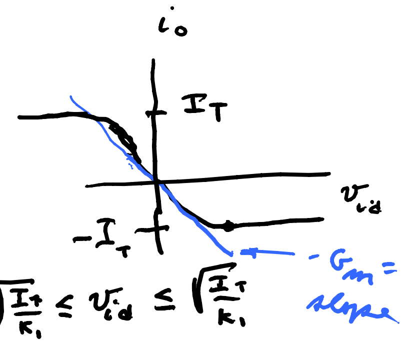
$$\frac{d i_o}{d\left(\sqrt{\frac{K_1}{I_T}}v_{id}\right)} = -I_T\sqrt{2 - x^2} - I_T x \frac{\frac{1}{2}(-2x)}{\sqrt{2 - x^2}} = -I_T\frac{\left[2 - x^2 - \frac{2}{2}x^2\right]}{\sqrt{2 - x^2}}$$

$= 0$ @ $\; 2 - 2x^2 = 0 \quad x^2 = 1$

find x for this $= 0$:

$$x^2 = \frac{K_1}{I_T} v_{id}^2 = 1 \Rightarrow v_{id}\Big|_{\cancel{max}\ min} = +\sqrt{\frac{I_T}{K_1}}$$

$$i_o\Big|_{\cancel{max}\ min} = -I_T\sqrt{\frac{K_1}{I_T}}\sqrt{\frac{I_T}{K_1}}\cdot\sqrt{2-1} = -I_T$$

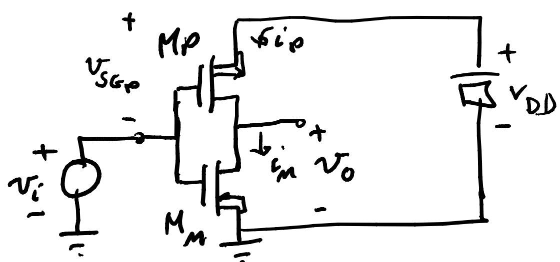


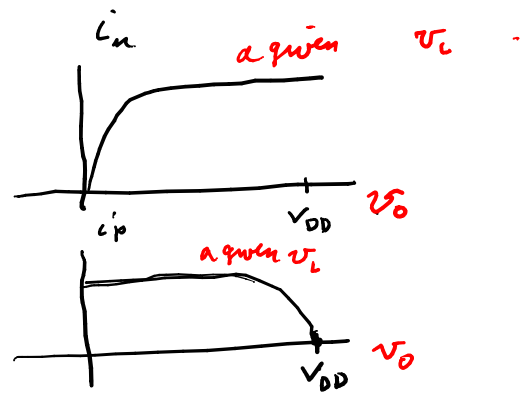
$$i_o = \begin{cases} -I_T\sqrt{\frac{K_1}{I_T}}\,v_{id}\sqrt{2-\left(\sqrt{\frac{K_1}{I_T}}\cdot v_{id}\right)^2} & \text{for } -\sqrt{\frac{I_T}{K_1}} \le v_{id} \le \sqrt{\frac{I_T}{K_1}} \\ -I_T & \text{for } \sqrt{\frac{I_T}{K_1}} \le v_{id} \\ +I_T & \text{for } v_{id} \le -\sqrt{\frac{I_T}{K_1}} \end{cases}$$

$$\left|\frac{di_o}{dv_{id}}\Big|_{v_{id}=0}\right| = G_m \text{ of an OTA} = \text{voltage to current gain for small signals}$$

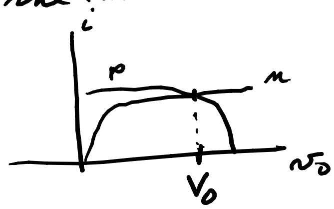
$$= \left|-I_T\cdot\sqrt{\frac{K_1}{I_T}}\sqrt{2}\right| = I_T\sqrt{\frac{2K_1}{I_T}} = \sqrt{2K_1 I_T}, \quad K_1 = \frac{KP}{2}\frac{W}{L}$$

Inverter behavior: pp. 336–346



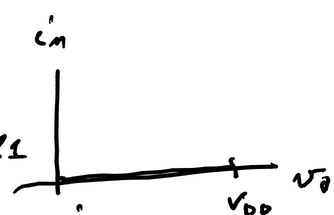


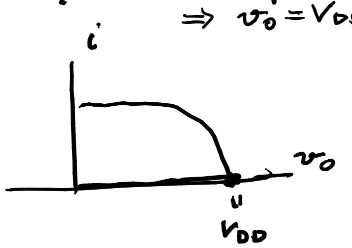
as $i_n = i_p$ the intersection gives v_o

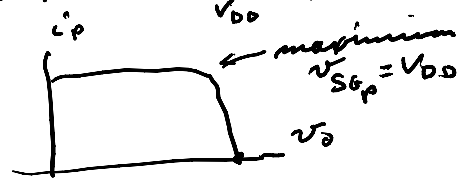


case 1: $v_i = 0 \Rightarrow$ digital 0

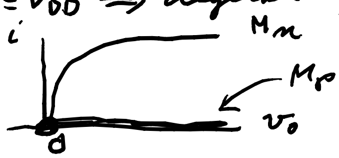
$\Rightarrow v_o = V_{DD} \Rightarrow$ digital 1



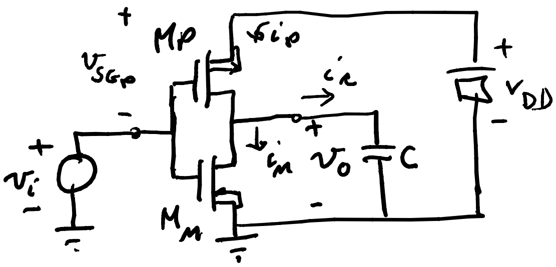


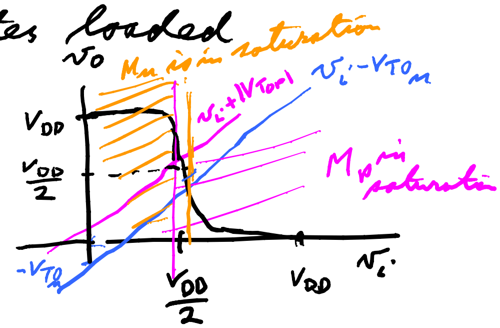


case 2: $v_i = V_{DD} \Rightarrow$ digital 1 $\Rightarrow v_o = 0 \Rightarrow$ digital 0



Next pulse v_i with the inverter loaded





In here

M_n is in saturation when

$v_i - V_{TO_n} \le v_O$

M_p is in saturation when

$v_{SG_p} - |V_{TO_p}| = V_{DD} - v_i - |V_{TO_p}| \le V_{DD} - v_O = v_{SD_p}$

$v_i + |V_{TO_p}| \ge v_O$

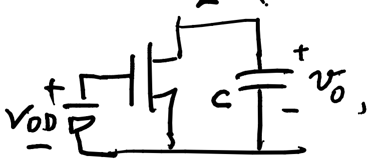
Derive delay time, eq. (4.156)

assume v_o = digital 1 = V_{DD} & $v_i\big|_{t<0} = 0$

and change v_i instantaneously to a digital 1, $v_i = V_{DD}$

this turns off Mp

$i_n = -C\frac{dv_o}{dt}$



$v_O(0) = V_{DD}$

@ $t=0$, $i_n = K_1(v_{GS} - V_{TO_n})^2 = K_1(V_{DD} - V_{TO_n})^2$

$$C\frac{dv_o}{dt} = -K_1(V_{DD} - V_{TO_n})^2 \Rightarrow v_o(t) = -\frac{K_1}{C}(V_{DD} - V_{TO_n})^2 \cdot t + v_o(0)$$

as $\int_0^t \frac{dv_o(\tau)}{d\tau}d\tau = v_o(t) - v_o(0)$

valid until $v_o = v_i - V_{TO_n}$