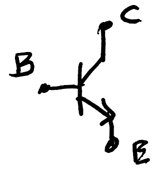
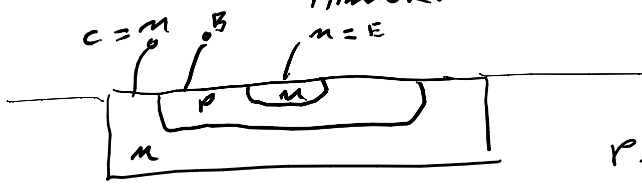
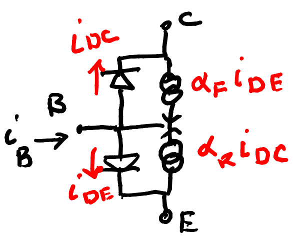
p. 387 = Ebers-Moll Model

EE 303H
09/24/09
(added notes 09/25)

Hmwk's due on Th

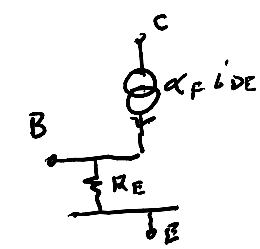




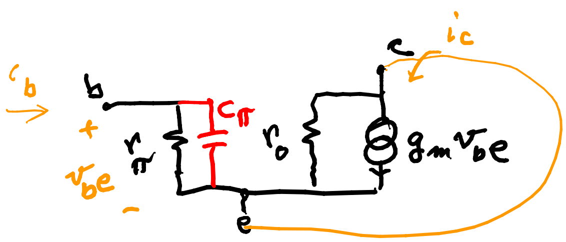


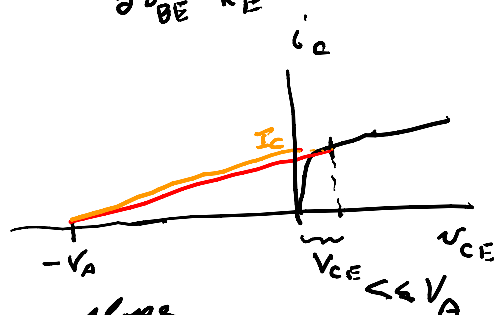
for small signal back bias B–C
forward bias B–E

$\Rightarrow$



$$-\frac{\partial i_E}{\partial v_{BE}} = \frac{1}{R_E}$$





g_o = slope
$$= \frac{I_c}{V_A + V_{CE}} \approx \frac{I_c}{V_A}$$

$$g_m = I_c / V_T$$

$$g_\pi = \frac{I_c}{V_T} \cdot \frac{1}{\beta} = \frac{g_m}{\beta}$$

$$g_\pi = -\frac{\partial i_E}{\partial v_{BE}}$$

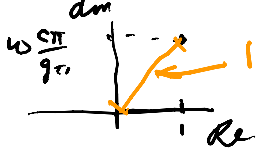
$$\left.\frac{i_c}{i_b}\right|_{v_{CE}=0} \Rightarrow$$

$$i_b = (g_\pi + s C_\pi)\, v_{be}$$

$$i_c = g_m v_{be}$$

$$\therefore \frac{i_c}{i_b} = \frac{g_m}{g_\pi + s C_\pi} = \frac{g_m/g_\pi}{1 + s\, C_\pi/g_\pi}$$ evaluate @ $s = j\omega$

Frequency response $$\left|\frac{i_c}{i_b}(j\omega)\right| = \frac{\beta}{\left|1 + j\omega \frac{C_\pi}{g_\pi}\right|} = \frac{\beta}{\sqrt{1 + \omega^2 \frac{C_\pi^2}{g_\pi^2}}}$$



$$\left|1 + j\frac{\omega C_\pi}{g_\pi}\right|$$ = magnitude of current gain

when $\left|\frac{i_c}{i_b}(j\omega)\right| = 1$ stops amplifying, that frequency is f_T; $\omega_T = 2\pi f_T$

$$\beta^2 = 1 + \omega_T^2 \frac{C_\pi^2}{g_\pi^2}; \qquad \omega_T = \sqrt{\frac{\beta^2 - 1}{C_\pi^2/g_\pi^2}}$$

f_T is specified in data sheets

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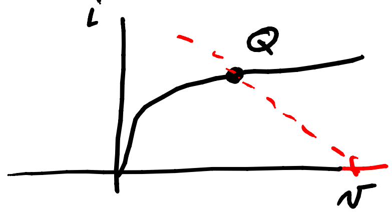
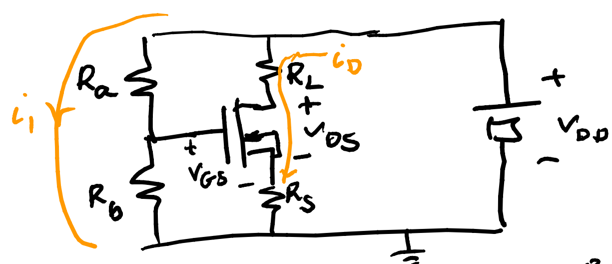
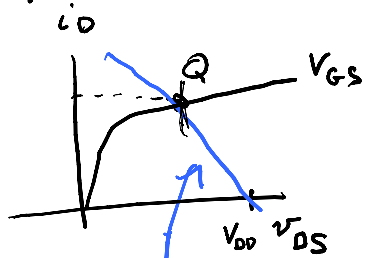
Biasing; bias is the quiescent point = Q point (at DC)



For an MOS transistor we want to fix V_{GS}, V_{DS}





$$i_D = \frac{V_{DD} - V_{DS}}{R_L + R_S}$$

1) $V_{DD} = (R_a + R_b)\, i_1$

2) $V_{DD} = (R_L + R_S)\, i_D + V_{DS}$

3) $V_{GS} = R_b i_1 - R_S i_D$

here R_S is such that $V_{GS}\downarrow$ if $I_D\uparrow$ or stabilizes to changes in I_D

1) solved for $i_1 \to 3$ $\quad V_{GS} = \dfrac{R_b \cdot V_{DD}}{R_a + R_b} - R_S i_D = \dfrac{V_{DD}}{1 + \frac{R_a}{R_b}} - R_S i_D$

Given V_{GS} & I_D & R_S $\Rightarrow$ fixes $\frac{R_a}{R_b}$ if know R_S

solve for $\frac{R_a}{R_b}$ $\Rightarrow$ $\left(1 + \frac{R_a}{R_b}\right)(V_{GS} + R_S I_D) = V_{DD}$

$$\frac{R_a}{R_b} = -1 + \frac{V_{DD}}{V_{GS} + R_S I_D}$$

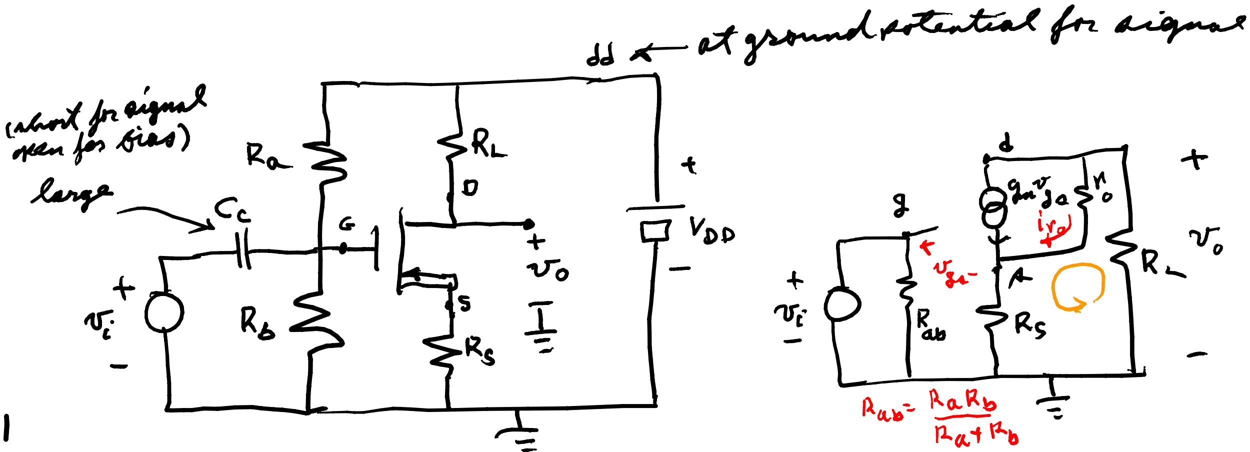
$\therefore$ $\frac{R_a}{R_b}$ will be known (can choose one of R_a or R_b choose one large to keep small bias power)

This means have to use an R_L from the above equation:

$$(R_L + R_S) I_D = V_{DD} - V_{DS}$$

$$R_L = -R_S + \frac{V_{DD} - V_{DS}}{I_D}$$

This now biases the transistor



1

desire $\frac{v_o}{v_i}$:

$$v_{gs} = v_i - R_S(g_m v_{gs} + i_{r_o}) \qquad 1)$$

$$v_o = r_o i_{r_o} + R_S(g_m v_{gs} + i_{r_o}) \qquad 2)$$

$$v_o = -R_L(g_m v_{gs} + i_{r_o}) \qquad 3)$$

from 2):

$$v_o = (r_o + R_S) i_{r_o} + R_S g_m v_{gs}$$

$$i_{r_o} = \frac{v_o - R_S g_m v_{gs}}{r_o + R_S} \qquad 2')$$

into 1)

$$v_{gs} = v_i - R_S g_m v_{gs} - \frac{R_S}{r_o + R_S}(v_o - R_S g_m v_{gs})$$

$$\left(1 + R_S g_m - \frac{R_S^2 g_m}{r_o + R_S}\right) v_{gs} = v_i - \frac{R_S}{r_o + R_S} v_o$$

If $R_S = 0$ then $i_{r_o} = \frac{v_o}{r_o} \Rightarrow$ from 3) $v_o = -R_L g_m v_{gs} - \frac{R_L}{r_o} v_o$

$$\left(1 + \frac{R_L}{r_o}\right) v_o = -R_L g_m v_{gs}$$

$$\left.\frac{v_o}{v_i}\right|_{\text{when } R_S = 0} = \frac{v_o}{v_{gs}} = \frac{-R_L g_m}{1 + \frac{R_L}{r_o}} = \frac{v_o}{v_i} \Rightarrow$$

ideal gain is $-R_L g_m$

--- notes added 09/25/09

for the general case, $G_L = 1/R_L$

i_{r_o} of 3) = i_{r_o} of 2): $i_{r_o} \overset{3)}{=} -G_L v_o - g_m v_{gs} = \frac{v_o}{r_o + R_S} - \frac{R_S}{r_o + R_S} g_m v_{gs} \overset{2)}{=} i_{r_o}$

4) $\Rightarrow v_{gs} = \frac{-\left(G_L + \frac{1}{r_o + R_S}\right) v_o}{g_m\left(1 - \frac{R_S}{r_o + R_S}\right)} = -\frac{(1 + G_L[r_o + R_S])}{r_o g_m} v_o$

4) into i_{r_o} of 3) $\Rightarrow i_{r_o} = -G_L v_o + \left(\frac{G_L + \frac{1}{r_o + R_S}}{\frac{r_o}{r_o + R_S}}\right) v_o = \left(-G_L + \frac{G_L(r_o + R_S) + 1}{r_o}\right) v_o = \frac{(1 + G_L R_S) v_o}{r_o}$

5) $\therefore\ i_{r_o} = \dfrac{(1+G_L r_S)\, v_o}{r_o}$

from 4) & 5) into 1):

$$(1+g_m R_S) v_{gs} \overset{4)}{=} -(1+g_m R_S)\frac{(1+G_L[r_o+R_S])}{r_o g_m} v_o \overset{1)}{=} v_i - R_S\, i_{r_o} \overset{5)}{=} v_i - \frac{R_S(1+G_L R_S) v_o}{r_o}$$

$$\Rightarrow \left\{ \frac{\cancel{R_S} + \cancel{G_L R_S^2}}{r_o} - \frac{[1 + g_m \cancel{R_S} + G_L r_o + G_L R_S + g_m R_S G_L r_o + \cancel{g_m G_L R_S^2}]}{r_o g_m} \right\} v_o = v_i$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{-g_m r_o}{1 + G_L(r_o + R_S + g_m R_S r_o)} = \frac{-g_m R_L}{g_o(R_L + R_S) + (1 + g_m R_S)}$$

If $R_S \to 0$ $\quad \left.\dfrac{v_o}{v_i}\right|_{R_S=0} = \dfrac{-g_m R_L}{1+g_o R_L}$ $\qquad$ & if also $g_o \to 0$ $\quad \left.\dfrac{v_o}{v_i}\right|_{R_S=g_o=0} = -g_m R_L$

(theoretical highest achievable voltage gain)