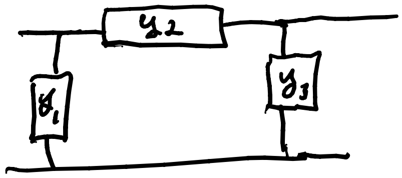
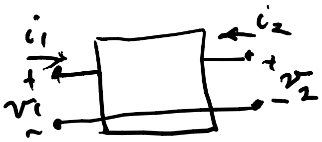
P.487 = BJT small signal "hybrid" pi

EE 303 H
09/22/09

Y matrix = appendix B (admittance)

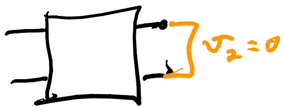
π equivalent circuit

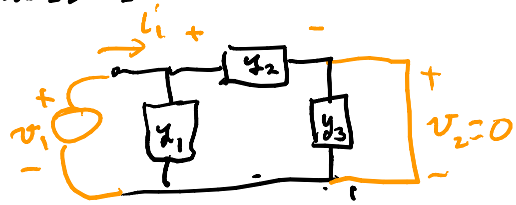



$$\begin{bmatrix} i_1\\ i_2\end{bmatrix} = \begin{bmatrix} y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}\begin{bmatrix} v_1\\ v_2\end{bmatrix}$$

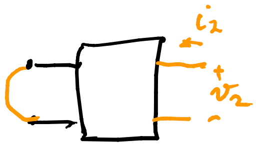
$$y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0} = y_1 + y_2$$

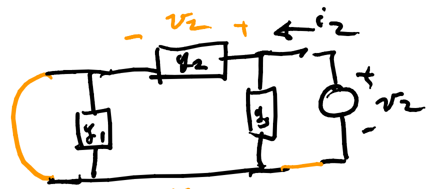



$y_1 = y_{11} - y_2$
$= y_{11} + y_{12}$

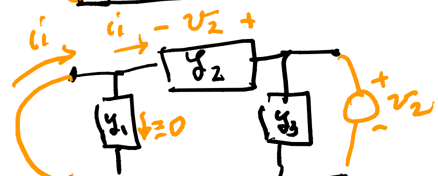
$$y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0} = y_3 + y_2$$



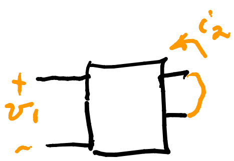



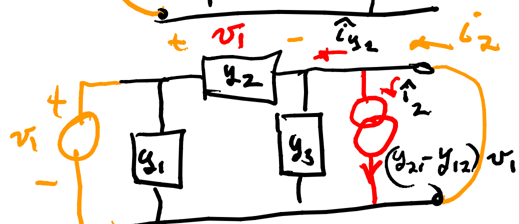

$y_2 = -y_{12}$

$$y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0} = -y_2$$

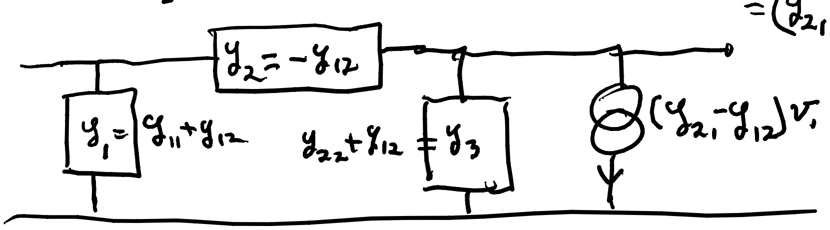

$$y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$$

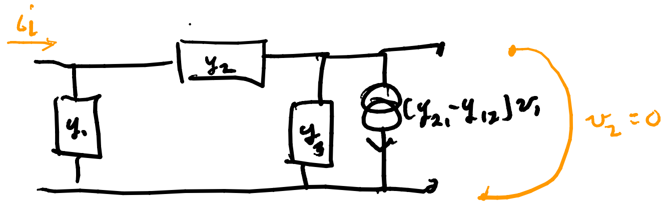



$= -y_2 \Rightarrow y_{21} = y_{12}$ if reciprocal
but $y_{21} \neq y_{12}$ in general
(for transistor)

$$i_2 = \hat{i}_{y_2} + \hat{i}_2 = -y_2 \cdot v_1 + \hat{i}_2 = y_{21} v_1 \Rightarrow \hat{i}_2 = y_{21}v_1 + y_2 v_1$$
$$= (y_{21} - y_{12})v_1$$






$$y_{12}=\frac{i_1}{v_2}\Big|_{v_1=0}$$

$$=-y_2$$

$$y_{11}=\frac{i_1}{v_1}\Big|_{v_2=0}=y_2\frac{v_1}{v_1}+y_1\frac{v_1}{v_1}=y_1+y_2$$

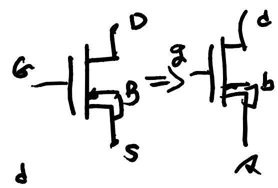
$$y_{22}=\frac{i_2}{v_2}\Big|_{v_1=0}$$

$$=y_2+y_3$$

has no current in the dependent source

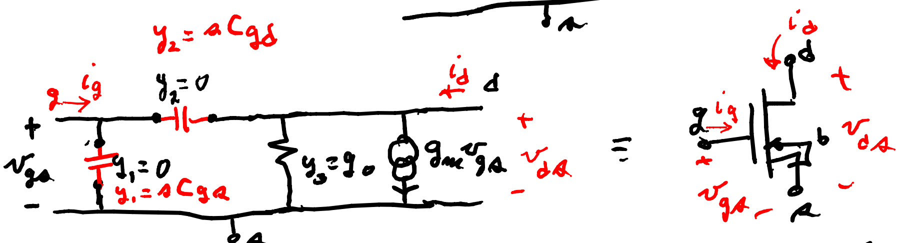
$\pi$ equivalent for the MOS

if in saturation NMOS $i_D=\frac{KP}{2}\frac{W}{L}(v_{GS}-VTO_n)^2(1+\lambda v_{DS})$




$$\frac{\partial i_D}{\partial v_{GS}}\Big|_Q=2\frac{KP}{2}\frac{W}{L}(V_{GS}-VTO_n)(1+\lambda V_{DS})=g_m$$

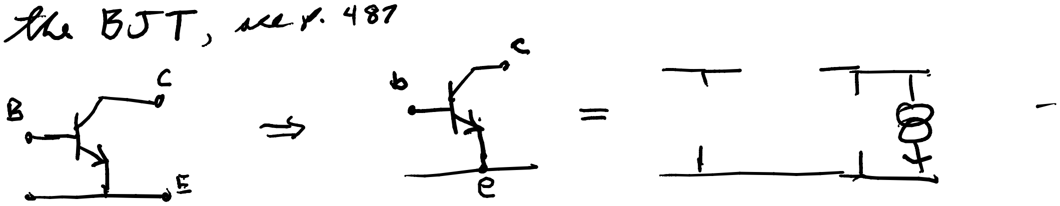
$$\frac{\partial i}{\partial v_{DS}}\Big|_Q=\lambda\frac{KP}{2}\frac{W}{L}(V_{GS}-VTO_n)^2=g_o$$


is the $\pi$-equivalent for the MOS transistor (small signal, linearized)

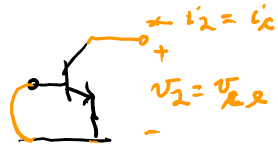
$$Y(s)=\begin{bmatrix} sC_{gs}+sC_{gd} & -sC_{gd} \\ -sC_{gd}+g_m & g_o+sC_{gd} \end{bmatrix}$$

Do for the BJT, see p. 487

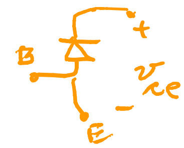


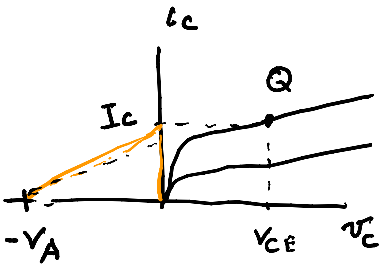

$i_c=\beta i_B$ , when bias we usually forward bias the B to E diode & back bias the B to C diode

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \left.\frac{i_c}{v_{ce}}\right|_{Q,\ v_1=0}$$

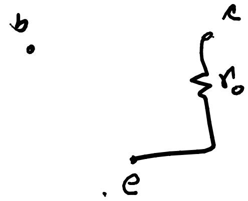



⇒

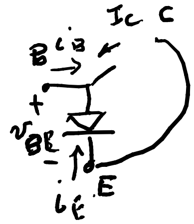


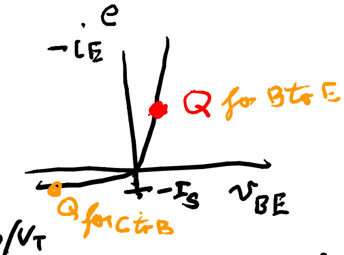




slope is $= \frac{I_c}{V_A} = \left.\frac{\partial i_c}{\partial v_{ce}}\right|_Q = g_o = \frac{1}{r_o}$




$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \left.\frac{i_B}{v_{BE}}\right|_Q$$




$\frac{\partial(-i_E)}{\partial v_{BE}}$ = slope of diode curve at Q

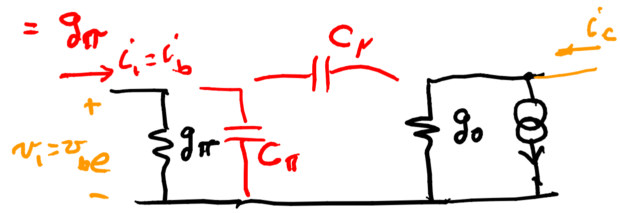
$i_D \approx I_S e^{v_D/V_T}$ (as forward bias can ignore $-I_S$ as it is very small)

$$\frac{\partial(-i_E)}{\partial v_{BE}} = \frac{\partial i_D}{\partial v_D} = \left.\frac{I_S}{V_T} e^{v_D/V_T}\right|_Q = \frac{-I_E}{V_T}$$

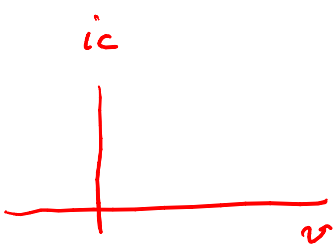
need $\left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q$ for $y_{11}$

$$i_c = \beta i_B = -\alpha i_E \Rightarrow i_B = -\frac{\alpha}{\beta} i_E \ ; \quad \underbrace{\frac{\partial(-i_E)}{\partial v_{BE}}}_{\frac{-I_E}{V_T}} = \underbrace{\frac{\partial(-i_E)}{\partial i_B}}_{+\frac{\beta}{\alpha}} \cdot \underbrace{\frac{\partial i_B}{\partial v_{BE}}}_{\text{desire}}$$

$$y_{11} = \frac{\partial i_B}{\partial v_{BE}} = \frac{-I_E}{V_T} \cdot \frac{\alpha}{\beta} = \frac{I_c}{V_T} \cdot \frac{1}{\beta} = g_\pi$$




⇒ π equivalent for a BJT




$$(y_{21} - y_{12}) v_1 = \beta g_\pi \cdot v_{be} = \frac{I_c}{V_T} \cdot v_{be}$$

$\approx y_{21} v_1 = y_{21} \cdot v_{be}$

$$= g_m v_{be}$$

$$g_m = I_c / V_T$$

$$g_\pi = \frac{I_c}{V_T} \cdot \frac{1}{\beta} = \frac{g_m}{\beta}$$

$$y_{21} = \left.\frac{\partial i_2}{\partial v_1}\right|_Q = \left.\frac{i_c}{v_{BE}}\right|_Q = \frac{i_c}{i_b} \cdot \frac{i_b}{v_{be}} = \beta \cdot g_\pi$$