EE 303H Fall 2009 Final Solutions

#1. a) $I_{C_{Q_1}} = I_{C_{Q_2}} = 2mA \Rightarrow \underline{I_1 = I_{C_{Q_1}} + I_{C_{Q_2}} = 4ma}$

as $\beta = 100$ & $I_C = \beta I_B \Rightarrow I_{B_{Q_1}} = I_{B_{Q_2}} = \frac{1}{\beta} \times \frac{I_1}{2} = \frac{2\times10^{-3}}{10^2} = 20\times10^{-6}$

∴ $\underline{I_2 = I_{B_{Q_1}} = 20\mu A}$ (also $= I_{B_{Q_2}}$)

For I_3: $I_3 = -I_{E_{Q_1}} - I_{B_{Q_2}}$ since $I_B + I_C + I_E = 0 \Rightarrow -I_E = I_B + I_C$

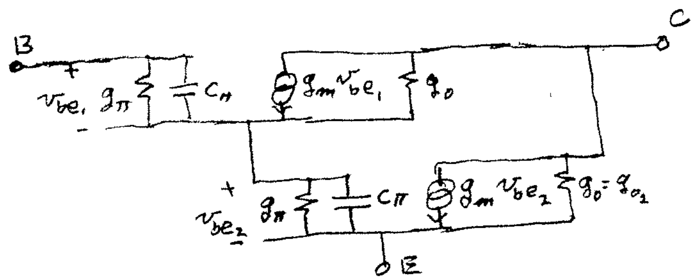
$= +(I_{C_{Q_1}} + I_{B_{Q_1}}) - I_{B_{Q_2}}$

$= I_{C_{Q_1}} = \underline{2ma = I_3}$

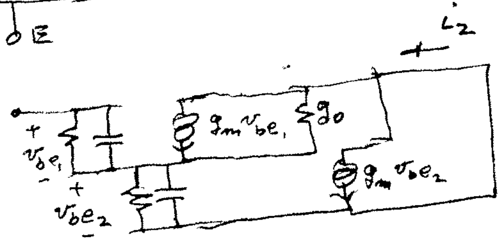
b) $g_m = \frac{I_C}{V_T} = \frac{2\times10^{-3}}{26\times10^{-3}} = \frac{1}{13}$ $\qquad g_\pi = \frac{g_m}{\beta} = \frac{1/13}{100}$

$g_o = \frac{I_C}{V_A} = \frac{2\times10^{-3}}{100} = 2\times10^{-5}$

c)



d) $y_{21} = \frac{i_2}{v_1}\Big|_{v_2 = 0 = short} \Rightarrow$



$v_1 = v_{be_1} + v_{be_2}$, $i_2 = g_m v_{be_1} + g_m v_{be_2} + g_o(-v_{be_2})$

$= g_m(v_{be_1} + v_{be_2}) - g_o v_{be_2} = g_m v_1 - g_o v_{be_2}$ (*)

To get v_{be_2}: For Q_1, $i_1 = y_\pi v_{be_1}$, $y_\pi = g_\pi + sC_\pi$

For Q_2: $i_1 + (g_m v_{be_1} + g_o(-v_{be_2})) = y_\pi v_{be_2}$

$= (y_\pi + g_m) v_{be_1} - g_o v_{be_2} \Rightarrow (g_o + y_\pi) v_{be_2} = (g_m + y_\pi) v_{be_1}$

as $v_{be_1} = v_1 - v_{be_2} \Rightarrow (g_o + y_\pi) v_{be_2} = (g_m + y_\pi) v_1 - (g_m + y_\pi) v_{be_2}$

$\Rightarrow (g_o + g_m + 2y_\pi) v_{be_2} = [g_m + y_\pi] v_1$ (**)

from (**) into (*)

$$\frac{i_1}{v_1} = g_m - \frac{g_o(g_m + y_\pi)}{g_o + g_m + 2y_\pi} = \frac{g_m^2 + (2g_m - g_o)(g_\pi + sC_\pi)}{g_o + g_m + 2(g_\pi + sC_\pi)} = y_{21}$$

#2 a) If $V_{in_1} > V_{in_2}$, the drains are on the left & sources on the right so $V_{GS} < 0$ for $M_1 \Rightarrow M_1$ off.
If $V_{in_2} > V_{in_1}$ then reverse 1 & 2 of previous $\Rightarrow M_2$ off by symmetry

b) $V_{in_1} > V_{in_2}$: M_1: $d = I_{n1}$, $s = out$ for M_2: $d = out$, $s = I_{n2}$
$V_{in_2} > V_{in_1}$: M_1: $d = out$, $s = I_{in1}$ for M_2: $d = I_{n2}$, $s = out$

c). As one transistor always has $I_D = 0$ then both have $I_D = 0$
But when $V_{in_1} > V_{in_2}$ then $V_{GS_2} > 0$ & M_2 has its I_D vs V_{DS} curve intersect the origin $\Rightarrow V_{DS_2} = 0 \Rightarrow V_{out} = V_{in_2} < V_{in_1}$
If $V_{in_1} < V_{in_2}$ then $V_{DS_1} = 0$ by symmetry $\Rightarrow V_{out} = V_{in_1} < V_{in_2}$
If $V_{in_1} = V_{in_2}$ then $V_{out} = V_{in_1} = V_{in_2}$, so

$$V_{out} = \min(V_{in_1}, V_{in_2})$$

#3. a) $g_m = \frac{\partial i_{out}}{\partial v_{in}} = \frac{\partial\, \alpha I_T \sinh(v_i/2V_T)}{\partial v_i} = \alpha I_T \cdot \frac{1}{2V_T} \cdot \left.\frac{\partial \sinh x}{\partial x}\right|_{x=0=v_i/2V_T}$

$\sinh x = \frac{1}{2}(e^{x} - e^{-x})$; $\frac{d \sinh x}{dx} = \frac{1}{2}(e^{x} + e^{-x}) = \cosh x$, $\cosh 0 = \frac{1}{2}(1+1) = 1$

$\therefore\ g_m = \alpha \frac{I_T}{2V_T}$

b). $i_1 = g_m v_1 \implies y_{11} = g_m,\ y_{12} = 0$

$i_2 = i_0 = g_m v_1 + sC\, v_0 = g_m v_1 + sC\, v_2 = y_{21} v_1 + y_{22} v_2$

$\therefore\ Y(s) = \begin{bmatrix} g_m & 0 \\ g_m & sC \end{bmatrix}$

4. as no γ is given, assume no body effect, that is $\gamma = 0$

If port 2 is open, $i_2 = 0 \Rightarrow v_{Z_4} = 0 \Rightarrow v_{Z_3} = 0 \Rightarrow i_{Z_3} = 0 \Rightarrow i_{Z_2} = 0 \Rightarrow i_{Z_1} = 0 \Rightarrow v_{Z_1} = 0$

$\Rightarrow i_1 = 0 \quad \Rightarrow \quad y_{in}(s) \equiv 0$

If port 2 is shorted, $v_{Z_3} + v_{Z_4} = 0 \Rightarrow v_{Z_1} + v_{Z_2} = 0 \Rightarrow$ input open $\Rightarrow Z_{in}(s) = 0$