EE 610
11/19/08

Maximally flat approximation, low pass

Rational approximation $\quad T(s) = \dfrac{K}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$

$$= \frac{K/a_0}{\frac{1}{a_0}s^n + \dots + 1}$$

let $\hat{s}^n = \frac{1}{a_0}s^n$ and divide $T(s)$ by K/a_0

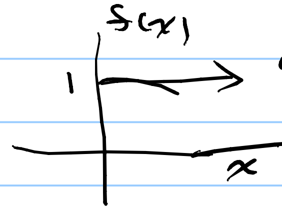
$$\hat{T}(s) = \frac{1}{\hat{s}^n + \hat{a}_{n-1}\hat{s}^{n-1} + \dots + 1}$$

$\therefore$ assume $\quad T(s) = \dfrac{1}{s^n + a_{n-1}s^{n-1} + \dots + a_1 s + 1}$

maximally flat magnitude at $\omega = 0$:

$$|T(s)|\Big|_{s=j\omega} \Rightarrow |T(j\omega)|^2 = T(s)T^*(s)\Big|_{s=j\omega} = T(s)T(-s)\Big|_{s=j\omega}$$

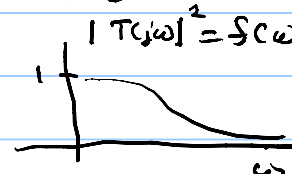
which is even in s



as many derivatives as possible $= 0$

$$f(\omega) = F(\omega^2)$$

$$= \frac{1}{\omega^{2n} + b_{2n-2}\,\omega^{2n-2} + \dots + b_2\omega^2 + 1}$$



desire as many derivatives as possible at $\omega = 0$ to be zero

$$f(\omega) = \frac{1}{g(\omega)} \Rightarrow \frac{d\,f(\omega)}{d\omega} = -\frac{1}{g(\omega)^2}\cdot\frac{dg(\omega)}{d\omega}$$

shows if $dg/d\omega = 0 \iff df(\omega)/d\omega = 0$

look at $g(\omega) = \omega^{2n} + b_{2n-2}\omega^{2n-2} + \cdots + b_2\omega^2 + 1$

to set derivatives to zero set all coefficients to zero

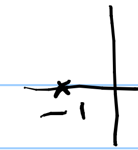
$$|T(\omega)|^2 = \frac{1}{\omega^{2n} + 1}$$ is maximally flat at $\omega = 0$

next find $T(s)$; $\left. T(s)T(-s) \right|_{s=j\omega} = \dfrac{1}{\omega^{2n}+1}$

$$T(s)T(-s) = \left. \frac{1}{\omega^{2n}+1} \right|_{\omega = s/j} = \frac{1}{(s/j)^{2n}+1} = \frac{1}{(-1)^n s^{2n}+1}$$

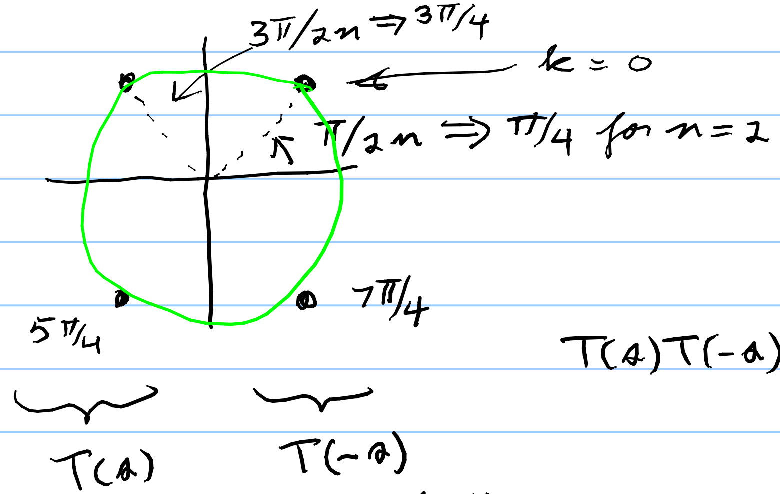
$\therefore$ need the roots of the polynomial $(-1)^n s^{2n} + 1 \Rightarrow 0$

1st do n even: $(-1)^n = +1 \Rightarrow s^{2n} + 1 = 0$; $s^{2n} = -1$



$$s^{2n} = e^{j\pi} = e^{j(\pi + 2\pi \times k)} \qquad k = 0, 1, \ldots$$

$$s_k = e^{j\left(\frac{\pi + 2k\pi}{2n}\right)} \qquad k = 0, 1, 2, \ldots, 2n$$



$$T(s)T(-s) = \frac{1}{s^{2n}+1}$$

for $n=2$: roots @ $s_k = e^{j\left(\frac{\pi + 2k\pi}{4}\right)}$

$k = 0 \Rightarrow s_0 = e^{j\pi/4} = \cos \pi/4 + j \sin \pi/4 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$k=1 \quad s_1 = -\frac{1}{\sqrt{2}} + j\,{}^1/\!\sqrt{2}\,, \quad s_2 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$$T(s) = \frac{1}{(s-s_1)(s-s_2)} = \frac{1}{\left(s+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)\left(s+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{1}{s^2 + \frac{2}{\sqrt{2}}s + 1}$$

$$T(s) = \frac{1}{s^2+\sqrt{2}\,s+1}$$

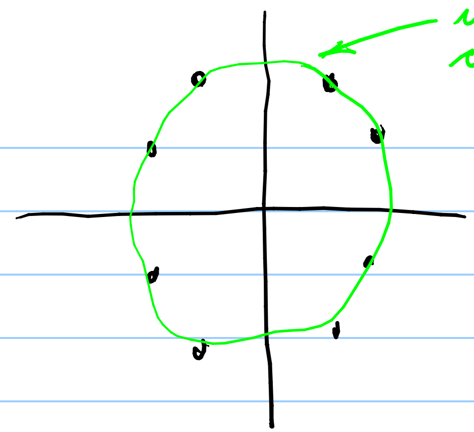
When n is odd, -1 is a root of $s^{2n}-1=0$

review

$$T(s)T(-s) = \frac{1}{(-1)^n s^{2n}+1} \quad \therefore \text{ if } n \text{ is odd desire roots of } (-1)s^{2n}+1=0$$

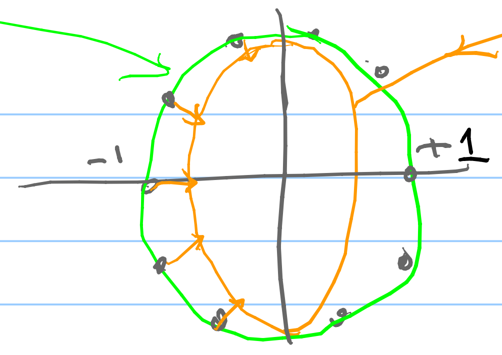
$n = \text{even}$

unit circle



$n = \text{odd}$

if use an ellipse get "equal ripple"

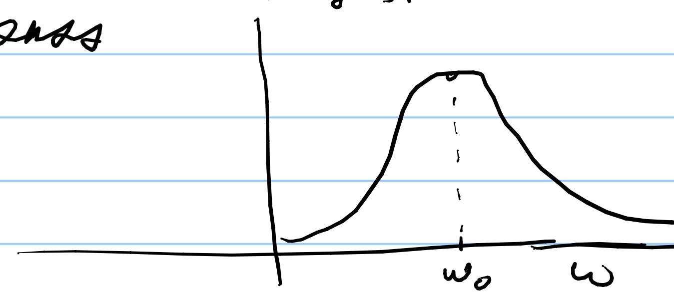


$$T(s) = \frac{1}{(s+1)(\cdots)}$$

no roots of $(-1)^n s^n + 1$
on the $j\omega$ axis

May desire bandpass

$|T(j\omega)|$



change frequency to p by

$$\frac{s}{\omega_0} = \frac{p}{\omega_0} + \frac{\omega_0}{p}$$

When $p = j\omega_0 \Rightarrow \dfrac{s}{\omega_0} = \dfrac{j\omega_0}{\omega_0} + \dfrac{\omega_0}{j\omega_0}$

$$= j1 - j1 = 0$$