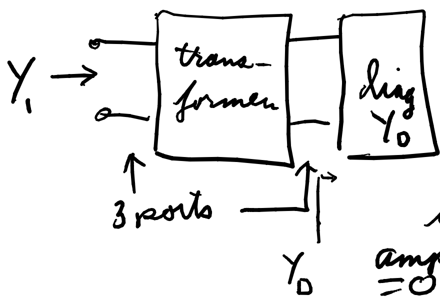
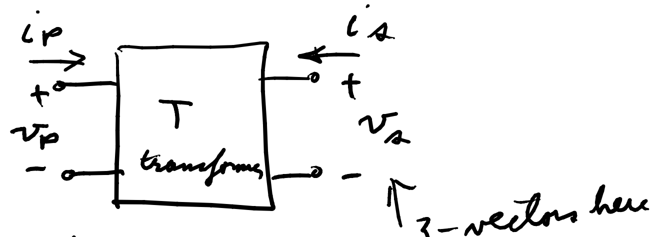
EE 610 W 11/05/08

Modified 11/07/08

$$Y = \begin{bmatrix} 3/2 & -1/4 & -9/8 \\ -1/4 & 0 & 0 \\ -\frac{9}{8} & 0 & 5/2 \end{bmatrix} + \begin{bmatrix} 0 & -1/4 & -1/8 \\ 1/4 & 0 & 0 \\ 1/8 & 0 & 0 \end{bmatrix} = Y_1 + Y_2$$





sum of amp turns $= 0$ $\rightarrow$ $i_s + T i_p = 0$

T is 3×3 here

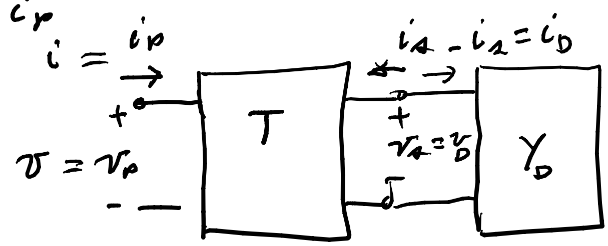
power in = power out $\rightarrow$ $v_p^T i_p = v_s^T i_s$

$$\Downarrow$$

$$v_p^T i_p = v_s^T \cdot T i_p$$

holds for any i_p $\Rightarrow$ $v_p^T = v_s^T T \Rightarrow v_p = T^T v_s$

Y_1 vs Y_D



$$i = Y_1 v = Y_1 v_p = Y_1 T^T v_s = Y_1 T^T v_D$$

$$\overset{\parallel}{i_p} \Rightarrow \quad T i_p = -i_s = i_D \Rightarrow T i_p = T i = T Y_1 T^T v_D = i_D$$

$$\Rightarrow \quad Y_D = T Y_1 T^T \Rightarrow \text{if } T^{-1}$$

$$\text{then} \quad Y_1 = T^{-1} Y_D \cdot T^{-T}$$

To transform:

$$T_1 Y_1 T_1^T = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1/4}{3/2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/2 & -1/4 & -9/8 \\ -1/4 & 0 & 0 \\ -9/8 & 0 & 5/2 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 & -1/4 & -9/8 \\ 0 & -\frac{(1/4)^2}{3/2} & (-9/8)(1/4)/3/2 \\ -9/8 & 0 & 5/2 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & 0 & -9/8 \\ 0 & -1/24 & -9/8 \times 1/6 \\ -9/8 & -\frac{9}{8}\cdot 1/6 & 5/2 \end{bmatrix}$$

$$T_2(T_1Y_1T_1^T)T_2^T = \begin{bmatrix}1&0&0\\0&1&0\\+\frac{9/8}{3/2}&0&1\end{bmatrix}\begin{bmatrix}3/2&0&-9/8\\0&-1/24&-3/16\\-9/8&-\frac{3}{16}&5/2\end{bmatrix}\begin{bmatrix}1&0&3/4\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}3/2&0&0\\0&-1/24&-3/16\\0&-3/16&5/2-\frac{(9/8)^2}{3/2}\end{bmatrix} \qquad = \frac{5\times16-27}{32} = \frac{80-27}{32} = \frac{53}{32}$$

$$T_3(T_2T_1Y_1T_1^TT_2^T)T_3^T = \begin{bmatrix}1&0&0\\0&1&0\\0&-\frac{(-3/16)}{-1/24}&1\end{bmatrix}\begin{bmatrix}3/2&0&0\\0&-1/24&-3/16\\0&-3/16&\cancel{5/2}\ \ 53/32\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&-9/2\\0&0&1\end{bmatrix}$$

$$\frac{3}{16}\cdot\frac{24}{1} = 9/2 \qquad 8,3 \qquad 8\times2$$

$$= \begin{bmatrix}3/2&0&0\\0&-1/24&0\\0&0&\frac{53}{32}-\left(\frac{9}{2}\times-\frac{3}{16}\right)\end{bmatrix} = \frac{80}{32}$$

$$16 \times 5 = 80$$

$$= \begin{bmatrix}3/2&0&0\\0&-1/24&0\\0&0&\frac{\cancel{107}\ 80}{32}\end{bmatrix} = \text{diagonal} = Y_D \qquad \frac{80-27+27}{32} = \frac{80}{32}$$

$$Y_1 = T_1^{-1}T_2^{-1}T_3^{-1}Y_DT_3^{-T}T_2^{-T}T_1^{-T}$$

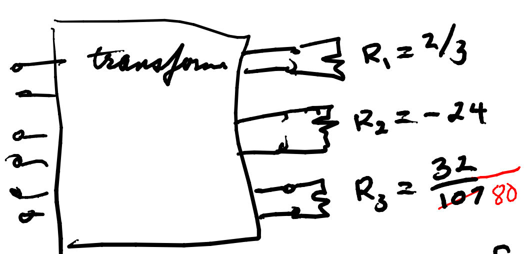
$$Y_1 = \begin{bmatrix}1&0&0\\-\frac{1}{6}&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-\frac{3}{4}&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&\frac{9}{2}&1\end{bmatrix}Y_D\begin{bmatrix}1&0&0\\0&1&9/2\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&-3/4\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&-1/6&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1&0&0\\-1/6&1&0\\-\frac{3}{4}&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&\frac{9}{2}&1\end{bmatrix}Y_D\begin{bmatrix}1&0&0\\0&1&9/2\\0&0&1\end{bmatrix}\begin{bmatrix}1&-1/6&-3/4\\0&1&0\\0&0&1\end{bmatrix}$$

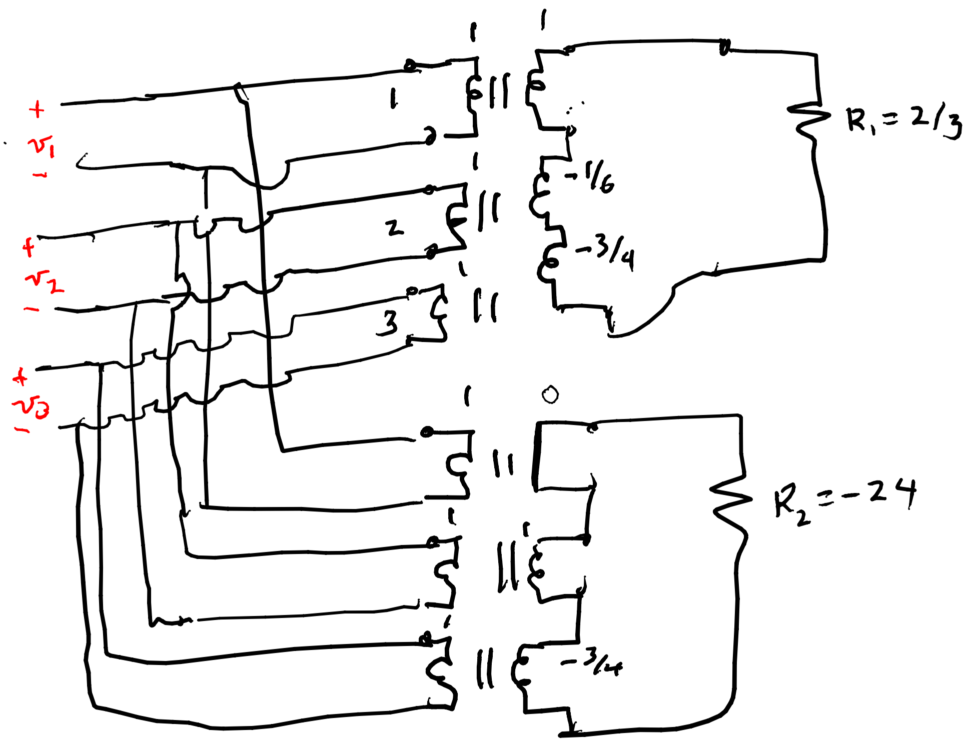
$$= \begin{bmatrix}1&0&0\\-1/6&1&0\\-3/4&9/2&1\end{bmatrix}Y_D\begin{bmatrix}1&-1/6&-3/4\\0&1&9/2\\0&0&1\end{bmatrix} = T^{-1}Y_DT^{-T} = Y_1$$

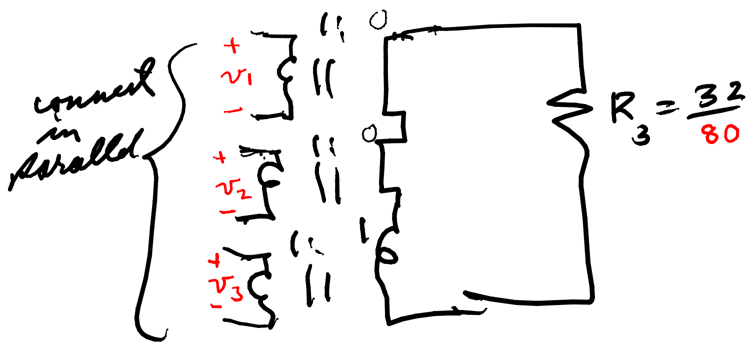
$$T^{-1}i_A + i_P = 0$$
$$v_A = T^{-T}v_P$$

$Y_1 \rightarrow$



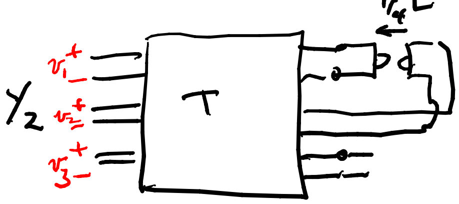
here $$\begin{bmatrix}1&0&0\\-1/6&1&0\\-3/4&9/2&1\end{bmatrix}\begin{bmatrix}i_{A_1}\\i_{A_2}\\i_{A_3}\end{bmatrix} = -\begin{bmatrix}i_{P_1}\\i_{P_2}\\i_{P_3}\end{bmatrix}; \quad \begin{bmatrix}v_{A_1}\\v_{A_2}\\v_{A_2}\end{bmatrix} = \begin{bmatrix}1&-1/6&-3/4\\0&1&9/2\\0&0&1\end{bmatrix}\begin{bmatrix}v_{P_1}\\v_{P_2}\\v_{P_3}\end{bmatrix}$$





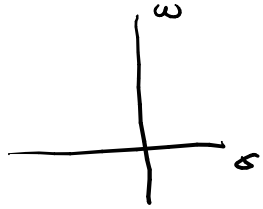
$$Y_2 = \begin{bmatrix} 0 & -1/4 & -1/8 \\ 1/4 & 0 & 0 \\ 1/8 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1/8}{1/4} & 1 \end{bmatrix} \begin{bmatrix} 0 & -1/4 & -1/8 \\ 1/4 & 0 & 0 \\ 1/8 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

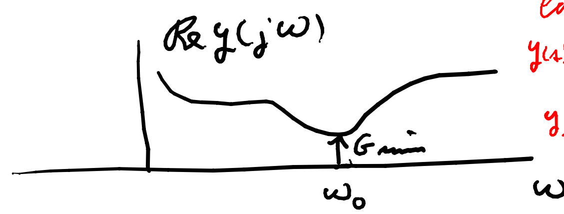
$$= \begin{bmatrix} 0 & -1/4 & 0 \\ 1/4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is rank 2 ⇒ 1 gyrator



put in parallel with Y_1 circuit & connect capacitors on ports 2 & 3

Bott–Duffin synthesis ⇒ uses no transformers
for PR $y(s)$ using Richards' function





can remove G_{min}

$y(s) \to y(s) - G_{min} = y_{min}(s)$

y_{min} is still PR

but $y_{min}(j\omega_0) = jB(\omega_0)$

∴ assume $y(s)$ has $\operatorname{Re} y(j\omega_0) = 0$; $y(j\omega_0) = 0 + jB(\omega_0)$

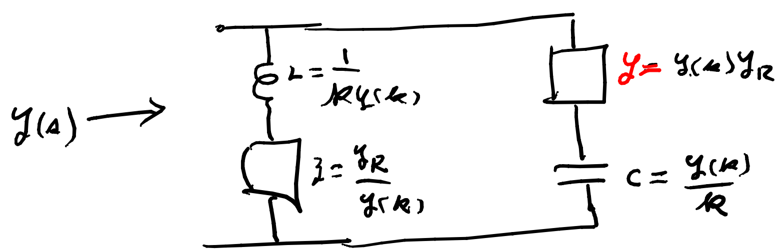
$$y_R = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} \Rightarrow k\,y(s)\,y_R - s\,y(k)\,y_R = k\,y(k) - s\,y(s)$$

$$y(s)\left[k\,y_R + s\right] = k\,y(k) + s\,y(k)\,y_R$$

$$y(s) = \frac{k\,y(k) + s\,y(k)\,y_R}{s + k\,y_R} = \frac{1}{\dfrac{s + k\,y_R}{k\,y(k)}} + \frac{1}{\dfrac{s + k\,y_R}{s\,y(k)\,y_R}}$$

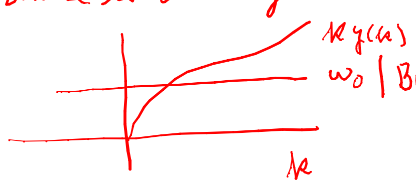
$$= \frac{1}{\dfrac{s}{k\,y(k)} + \dfrac{y_R}{y(k)}} + \frac{1}{\dfrac{1}{y(k)\,y_R} + \dfrac{k}{s\,y(k)}}$$

every term here is PR if $k > 0$



Note: This is a balanced bridge as the product of cross-arm admittances is $y(k)^2$

Next choose k so $y_R(j\omega_0) = 0$ or ∞ so can remove a $j\omega_0$ pole of $\frac{1}{y_R}$ or y_R which decreases the degree by 2 after the poles are removed. There is always a $k > 0$ to do this so y_R is PR



⇒ y_R is zero at ω_0 if $B(\omega_0) > 0$
y_R has a pole at ω_0 if $B(\omega_0) < 0$