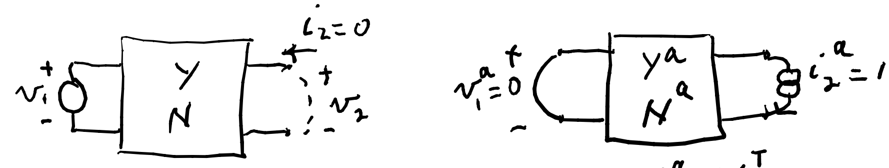
$\therefore$ set $i_2^a = 1 \Rightarrow \boxed{\Delta v_2 = v_b^{a\,T} \Delta Y v_b}$

$\Rightarrow$

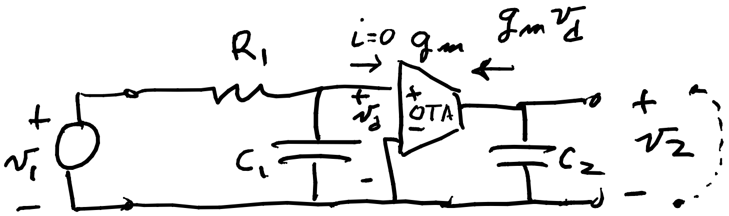


$Y^a = Y^T$

$S_x^{T(s)}$ = sensitivity of $T(s)$ = transfer function to the parameter x

= % change in $T(s)$ vs % change in x (in differential form)

$= \dfrac{x}{T(s)} \dfrac{dT(s)}{dx}$ will get $\dfrac{dT(s)}{dx}$ from the circuits using the adjoint

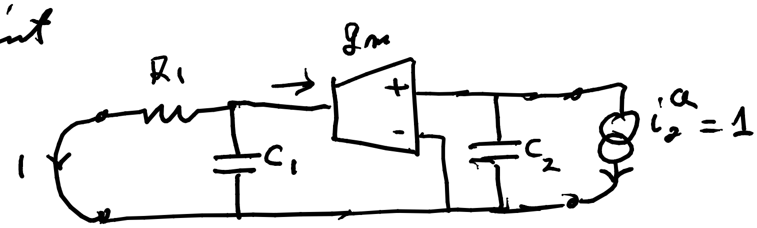


$Y_{OTA} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$

$Y_{OTA}^a = Y_{OTA}^T = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix}$

$\Downarrow$

$\Rightarrow$ adjoint



If we desire $S_{g_m}^{v_2/v_1}$; need $\dfrac{v_2}{v_1}(s)$ use $v_1 = 1v$

$$v_d = \frac{1/sC_1}{R_1 + \frac{1}{sC_1}} \cdot v_1 = \frac{1}{1+sR_1C_1}\cdot v_1 \;, \quad v_2 = -g_m v_d \cdot \frac{1}{sC_2}$$

$$v_2 = -\frac{g_m}{sC_2}\cdot\frac{1}{1+sR_1C_1}\cdot v_1 \Rightarrow \frac{v_2}{v_1} = \frac{-g_m}{sC_2(1+sR_1C_1)}$$

$$S_{g_m}^{v_2/v_1} = \frac{g_m}{v_2/v_1}\cdot\frac{d\, v_2/v_1}{d g_m} = \frac{g_m}{\frac{-g_m}{sC_2(1+sR_1C_1)}}\cdot\frac{-1}{sC_2(1+sR_1C_1)} = 1$$

$$Y_{b\times b} = \begin{array}{cc} \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 \\ 0 & sC_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & sC_2 \end{bmatrix} & \begin{array}{l} \leftarrow v_3 \\ \vdots \\ \\ \\ \leftarrow v_7 \end{array} \end{array} \qquad G_1 = 1/R_1$$

$\Delta Y_{b\times b}$ when change $g_m \to \Delta g_m$

$$\Delta Y_{b\times b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{desire } v_b^{aT} \Delta Y_{b\times b} v_b = v_b^a \Delta g_m v_b$$

$$\Delta v_2 = [v_{b_3}^a \; v_{b_4}^a \; v_{b_5}^a \; v_{b_6}^a \; v_{b_7}^a] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{b_3} \\ v_{b_4} \\ v_{b_5} \\ v_{b_6} \\ v_{b_7} \end{bmatrix} = v_{b_6}^a \cdot \Delta g_m \cdot v_{b_5}$$

$$\frac{\Delta v_2}{\Delta g_m} = v_{b_6}^a \cdot v_{b_5}$$

$$= \frac{\Delta(v_2/v_1)}{\Delta g_m} = \frac{d\left(\frac{v_2}{v_1}\right)}{d g_m}$$ ∴ solve N for v_{b_5} & N^a for $v_{b_6}^a$

but $v_{b_5} = v_d = \frac{1/sC_1}{R_1 + 1/sC_1} \cdot \overset{1}{\overset{\shortparallel}{v_1}} = \frac{1}{1+sR_1C_1}$

& $v_{b_6}^a = -\frac{1}{sC_2} \cdot \underset{1}{\underset{\shortparallel}{i_2^a}} = -\frac{1}{sC_2}$

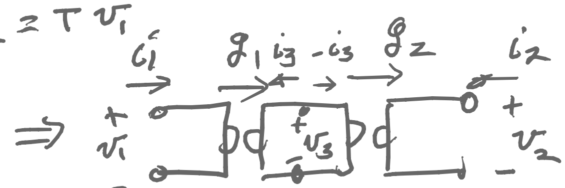
∴ by solving N & N^a we get $\frac{d\frac{v_2}{v_1}}{d g_m} = v_{b_6}^a \cdot v_{b_5} = \frac{-1}{sC_2} \cdot \frac{1}{1+sR_1C_1}$

(which was what got by direct calculation!)

If no admittance $Y_{b \times b}$ of the internal circuit can make replacements (usually can use gyrators), to form the adjoint

$i_1 \rightarrow$ 1:T $\leftarrow i_2 \Rightarrow i_1 + T i_2 = 0 \Rightarrow i_1 = -T i_2$

$v_2 = T v_1$



$$\begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & g_1 \\ -g_1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \end{bmatrix}, \quad \begin{bmatrix} -i_3 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g_2 \\ -g_2 & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_2 \end{bmatrix}$$

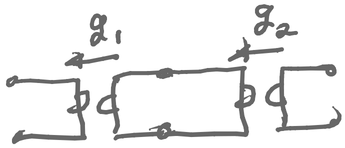
$i_1 = g_1 v_3$ & $i_2 = -g_2 v_3 \Rightarrow v_3 = -\frac{1}{g_2} i_2$

$\Rightarrow i_1 = -\frac{g_1}{g_2} i_2$ looks like $T = g_1/g_2$

$i_3 = -g_1 v_1$ & $-i_3 = g_2 v_2 =$

$\Rightarrow -g_1 v_1 = -g_2 v_2 \Rightarrow \frac{v_2}{v_1} = \frac{g_1}{g_2} = T$

the adjoint

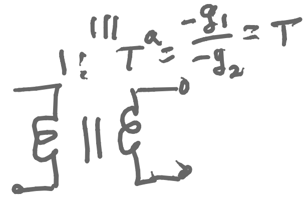


as $Y_{gyr} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$,

$Y^a_{gyr} = Y^T_{gyr} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$

= turned around gyrator

1 : $T^a = \frac{-g_1}{-g_2} = T$



Next: Design of transfer functions by setting up state-variable equations (do for single input single output); do for linear time-invariant; no pole at $\infty = s$

$$\dot{x} = Ax + Bu \qquad T(s) = \frac{Y(s)}{U(s)} = D + C(s1_n - A)^{-1}B$$
$$y = Cx + Du$$

$\therefore$ create C, B, D, A from $T(s)$ $\quad n = \#$ of state variable given $T(s) = \frac{y(s)}{u(s)} = d + \frac{\sum_{i=0}^{n-1} c_i s^i}{\sum_{i=0}^{n} a_i s^i}$ $\therefore$ know d, c_i, a_i

$i = 0, \dots, n-1$

$a_n = 1$

follow what is in the book, p. 370-371

Let $\frac{y_1(s)}{u(s)} = \frac{\sum_{i=0}^{n-1} c_i s^i}{\sum_{i=0}^{n} a_i s^i} = \frac{y_1}{x_0} \cdot \frac{x_0}{u(s)}$

let $\frac{y_1}{x_0} = \sum_{i=0}^{n-1} c_i s^i$, $\frac{u}{x_0} = \sum_{i=0}^{n} a_i s^i$

$$\Rightarrow \underbrace{y_1 = \sum_{i=0}^{n-1} c_i s^i x_0}_{\text{gives output C part}}$$

$$u = \sum_{i=0}^{n} a_i s^i x_0 = s^n x_0 + \sum_{i=0}^{n-1} a_i s^i x_0$$

$$\equiv \underbrace{s^n x_0 = u - \sum_{i=0}^{n-1} a_i s^i x_0 = s x_n}_{\text{gives } Ax + Bu \text{ part}}$$

set $s^i x_0 = x_{i+1}$; $x_0 = x_1$, $s x_0 = s x_1 = x_2$

$s^2 x_0 = s x_2 = x_3 \quad \dots \quad s^{n-1} x_0 = x_n$

use $s = d/dt$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$; $\frac{dx}{dt} = \begin{bmatrix} s x_1 \\ s x_2 \\ \vdots \\ s x_{n-1} \\ s x_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$

$$y = \begin{bmatrix} c_0 & c_1 & \cdots & c_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + d u$$