Look at $S(s)$ for $y_L(s)$:

$$S_{y_L} = \frac{1-y_L}{1+y_L} = \frac{1 - \dfrac{k f(k) - s f(s)}{k f(s) - s f(k)}}{1 + \dfrac{k f(k) - s f(s)}{k f(s) - s f(k)}} = \frac{k y(s) - s y(k) - k y(k) + s y(s)}{k y(s) - s y(k) + k y(k) - s y(s)}$$

$$= \frac{k+s}{k-s} \cdot \frac{(-y(k) + y(s))}{(y(k) + y(s))} = \frac{1+s/k}{1-s/k} \cdot \frac{1 - \dfrac{y(s)}{y(k)}}{1 + \dfrac{y(s)}{y(k)}} \times (-1)$$

$$\left| S_{y_L}(j\omega) \right| = \left| \frac{1 + j\omega/k}{1 - j\omega/k} \right| \left| S_y(j\omega) \right| \left| -1 \right| \leq 1$$

$$\left| \frac{1 + j\omega/k}{1 - j\omega/k} \right| = \frac{1+\omega^2/k^2}{1+\omega^2/k^2} = 1, \qquad \left| S_y(j\omega) \right| \leq 1, \qquad \left| -1 \right| = 1$$

$\therefore \; |S_{y_L}(j\omega)|^2 \leq 1 \Rightarrow S^*_{y_L}(s) S_{y_L}(s) \leq 1$ in $\sigma > 0$ if $S_{y_L}(s)$ is analytic

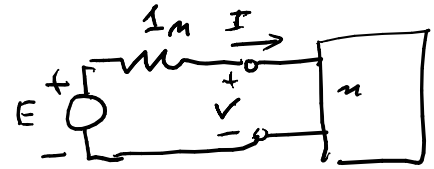
S_{y_L} has no poles in $\sigma > 0$ if S_y has no poles there
this is true if y is PR

But S_{y_L} looks to have a pole at $s = k$ due to $\frac{k+s}{k-s}$
so if y is PR, S_{y_L} is rational & $k-s$ cancels
in numerator & denominator.

$\therefore$ $S_{y_L}(s)$ is BR as is $S_y(s)$ if $y(s)$ is PR

Note $y(s) = \sqrt{s}$ is positive-real

Look at $S(s)$, $Y(s)$ as matrices



$$P_{in} = \operatorname{Re} V^{*T} I$$

$$2V^i = V + I = E$$

$$2V^r = V - I$$

$$\Rightarrow \quad V = V^i + V^r$$

$$I = V^i - V^r$$

$$P_{in} = \operatorname{Re}(V^{*T} I) = \operatorname{Re}\left((V^{iT*} + V^{rT*})(V^i - V^r)\right)$$

$$= V^{i*T}V^i - V^{rT*}V^r + \underbrace{\operatorname{Re}\left(V^{r*T}V^i - V^{i*T}V^r\right)}_{=0}$$

$$V^{r*T}V^i - V^{rT}V^{i*TT}$$

$$'' \quad \left(V^{r*T}V^i\right)^*$$

$$= V^{i*T}V^i - V^{iT*}S^{T*}SV^i$$

$$= V^{i*T}\left[1_n - S^{T*}S\right]V^i$$

If lossless $1_n = S^{T*}(j\omega)S(j\omega)$

$$= S^T(-j\omega)S(j\omega) \quad \text{for all real } \omega$$

can extend to all $s = \sigma + j\omega$ by analytic continuation $j\omega \to s$

if lossless $1_n = S^T(-s)S(s)$ for all s

$\therefore$ the inverse exists & is given by $S^{-1}(s) = S^T(-s)$

$$V = ZI, \quad I = YV$$

$$2V^i = V + I = V + YV = (1_n + Y)V$$

$$2V^r = V - I = V - YV = (1_n - Y)V = 2SV^i = S(1_n + Y)V$$

$$S = (1_n - Y)(1_n + Y)^{-1}$$

Ex: lossless $y(s) = sC$

$$S_y(s) = \frac{1 - sC}{1 + sC} = S_y(-s)^{-1} \text{ as } S_y(-s) = \frac{1 - (-s)C}{1 + (-s)C} = \frac{1 + sC}{1 - sC}$$

(has a pole in $\sigma > 0$)

Ideal transformers are lossless; $P_{in} \equiv 0$

$$i_2 + T i_1 = 0\,, \qquad v_1^{*T} i_1 + v_2^{*T} i_2 = v_1^{*T} i_1 + v_2^{*T}(-T i_1) = 0$$

$T =$ turns ratio $\qquad v_1^{*T} = T v_2^{*T} \Rightarrow \quad v_1 = T^{*T} v_2$

if conjugate

but for a real transformer $T = T^*$

$$\begin{bmatrix} T & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & -T \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

i.e. no Y or Z

no $YV = I$ or $ZI = V$

But S exists even though Z & Y do not

$$AV = BI\,; \qquad V = V^i + V^r, \quad I = V^i - V^r$$

$$A(V^i + V^r) = B(V^i - V^r) \Rightarrow \quad (A - B)V^i = -(A+B)V^r$$

$$\& \quad V^r = S V^i \Rightarrow \quad S = (B+A)^{-1}(B-A)$$

for the transformer

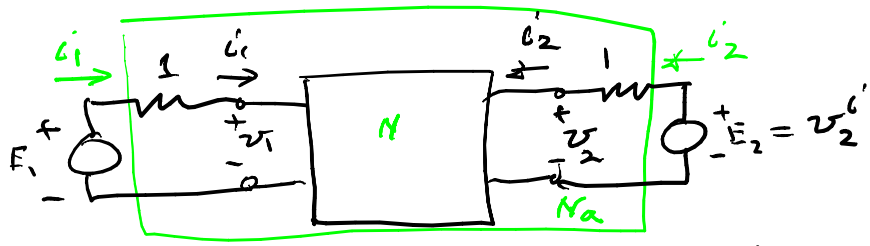
$$B = \begin{bmatrix} 0 & 0 \\ 1 & -T \end{bmatrix}, \quad A = \begin{bmatrix} T & 1 \\ 0 & 0 \end{bmatrix}, \quad A + B = \begin{bmatrix} T & 1 \\ 1 & -T \end{bmatrix}$$

$$(B+A)^{-1} = \frac{1}{-(1+T^2)} \begin{bmatrix} -T & -1 \\ -1 & T \end{bmatrix}, \qquad B - A = \begin{bmatrix} -T & -1 \\ 1 & -T \end{bmatrix}$$

$$S = (B+A)^{-1}(B-A) = \frac{-1}{1+T^2} \underbrace{\begin{bmatrix} -T & -1 \\ -1 & T \end{bmatrix} \begin{bmatrix} -T & -1 \\ 1 & -T \end{bmatrix}}_{\begin{bmatrix} T^2-1 & 2T \\ 2T & 1-T^2 \end{bmatrix}}$$

$$S_{trans} = \begin{bmatrix} \frac{1-T^2}{1+T^2} & -\frac{2T}{1+T^2} \\ \frac{-2T}{1+T^2} & \frac{T^2-1}{1+T^2} \end{bmatrix}$$

as the transformer is lossless, $S_{trans}^{-1} = S^T$



$E_2 = 0 \Rightarrow v_2^i = 0 = \frac{v_2 + i_2}{2} = v_2 = -i_2$; $E_1 = v_1^i$

$$\begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} \Rightarrow v_2^r = S_{21} \cdot v_1^i$$

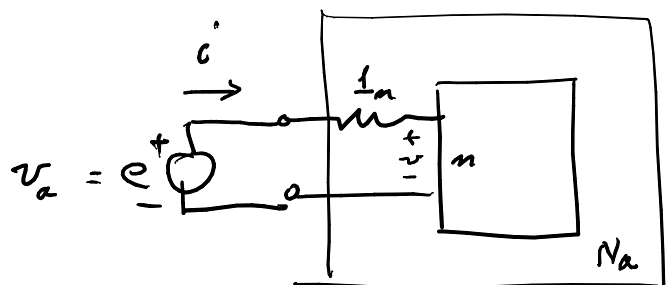
$$= S_{21} \cdot E_1$$

$$2 v_2^r = v_2 - i_2 = 2 v_2 \qquad = 2v_2$$

$S_{21} = 2\frac{v_2}{E_1}$ = terminated voltage gain

one method of design is from $S_{21}(j\omega) \to S_{21}(s) \to \underset{2\times 2}{S}$ which is para-unitary

$$S(s)\,S^T(-s) = 1_2$$



$i = Y_a \cdot e$

$v = e - i = e - Y_a e$

$2v^i = v + i = (e - Y_a e) + Y_a e = e$

$2v^r = v - i = (e - Y_a e) - Y_a e$

$= (1_n - 2Y_a)e$

$2v^i = e$

$2v^r = S(2v^i) = (1_n - 2Y_a)e$

$= (1_n - 2Y_a)(2v^i)$

$\Rightarrow \quad S = 1_n - 2Y_a$

note $Av = Bi \Rightarrow (1_n - Y_a)i = (1_n - Y_a)Y_a e$

$= (Y_a - Y_a^2)e = Y_a(1_n - Y_a)e$

$= Y_a v$

can set

$B = 1_n - Y_a$, $A = Y_a \Rightarrow S = (B+A)^{-1}(B-A)$

$= 1_n(1_n - 2Y_a)$

It is true Y_a exists when N is passive (since Y does not have −1 to cancel +1)