Richards' function p. 361 EE 610 W 10/08/08

Eq. (8.6-1) $R(s) = \dfrac{k f(s) - s f(k)}{k f(k) - s f(s)}$ is PR if $f(s)$ is PR for $k > 0$

note $1/f(s)$ is PR if $f(s)$ is PR
also then $1/R(s)$ is PR & $\hat{R}(s) = \dfrac{k\frac{1}{f(s)} - s\frac{1}{f(k)}}{k\frac{1}{f(k)} - s\frac{1}{f(s)}}$

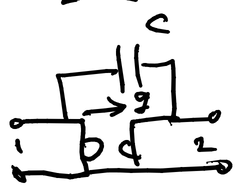
$$= \frac{k f(k) - s f(s)}{k f(s) - s f(k)} = \frac{1}{R(s)}$$

The zero & pole at $s = k$ cancel

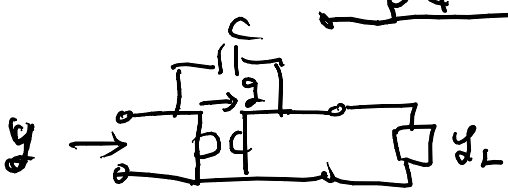
If k is a zero of the even part of $f(s)$ then $s = -k$ also cancels.

Then $\delta[R] = \delta[f(s)] - 1$

Look at



$$Y = \begin{bmatrix} sC & -sC + g \\ -sC - g & sC \end{bmatrix}$$



$$y = y_{11} - y_{12}(y_{22} + y_L)^{-1} y_{21}$$

$$= \frac{\Delta y + y_{11} y_L}{y_{22} + y_L} = \frac{g^2 + sC y_L}{sC + y_L}$$

$$sCy + y y_L = g^2 + sC y_L$$

$$(y - sC) y_L = g^2 - sCy \Rightarrow y_L = \frac{g^2 - sC y(s)}{y(s) - sC}$$

desire to equate y_L with a Richards' function

$$\hat{R}(s) = \frac{k f(k) - s f(s)}{k f(s) - s f(k)} \Rightarrow y_L(s) = \frac{g^2\left(1 - sC\frac{y(s)}{g^2}\right)}{y(s) - sC}$$

$$= k f(k)\left(\frac{1 - \frac{s}{k}\cdot\frac{f(s)}{f(k)}}{k\left(f(s) - \frac{s}{k}\, f(k)\right)}\right)$$

$$\hat{R}(s) = f(k)\left(\frac{1 - \frac{s}{k}\cdot\frac{f(s)}{f(k)}}{f(s) - \frac{s}{k}f(k)}\right)$$

for numerator: $\frac{s}{k}\cdot\frac{f(s)}{f(k)} = sC\,\frac{y(s)}{g^2}$

with $f(s) = y \Rightarrow \frac{C}{g^2} = \frac{1}{kf(k)}$

for denominator: $\frac{s}{k}f(k) = sC \Longrightarrow C = \frac{f(k)}{k} > 0$

This says that

$g = \pm f(k)$; $C = \frac{f(k)}{k}$

$$g^2 = C\cdot kf(k) = \frac{f(k)}{k}\cdot kf(k) = f(k)^2$$

We want

$$y_L(s) = g^2\left(\frac{1 - \frac{sC\cdot y(s)}{g^2}}{y(s) - sC}\right) \Rightarrow f(k)\left(\frac{1 - \frac{s}{k}\cdot\frac{f(s)}{f(k)}}{f(s) - \frac{s}{k}\cdot f(k)}\right)$$

$$= f(k)^2\left(\frac{1 - \frac{sC\,y(s)}{g^2}}{y(s) - sC}\right)$$

set $f(s) = y(s)$

set $\frac{y_L(s)}{f(k)} = \hat{R}(s)$

$\therefore$ can synthesize $y(s)$ by forming $\hat{R}(s)$ & scale by $f(k)$ to get the load $y_L(s) = f(k)\hat{R}(s)$

& $\delta[y_L(s)] = \delta[y] - 1$ if k satisfies

$$2\,\mathrm{Ev}\,f(s)\Big|_{s=k} = f(k) + f(-k) = 0$$

Ex: $y(s) = \frac{2s}{s^2+3}$ $\qquad y(s) + y(-s) \equiv 0$ can choose any $k > 0$

let $k = 1$ $\qquad y(s)\Big|_{s=k} = \frac{2}{4} = 1/2 = y(k)$ $\qquad f(s) = y(s) = \frac{2s}{s^2+3}$

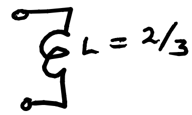
$$\hat{R}(s) = \frac{kf(k) - sf(s)}{kf(s) - sf(k)} = \frac{1\cdot\frac{1}{2} - s\left(\frac{2s}{s^2+3}\right)}{1\cdot\frac{2s}{s^2+3} - s\frac{1}{2}}$$

rewrite $\hat{R}(s) = \dfrac{\frac{1}{2}(s^2+3) - 2s^2}{2s - \frac{1}{2}s(s^2+3)} = \dfrac{(\frac{1}{2}-2)s^2 + 3/2}{-\frac{1}{2}s^3 + (2 - 3/2)s}$

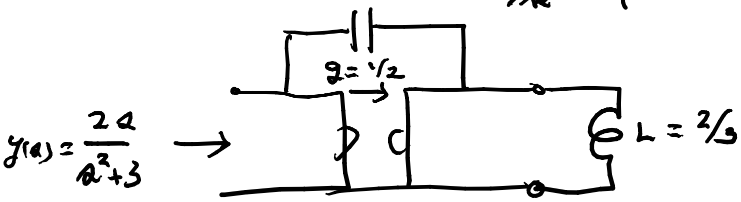
$$= \frac{-\frac{3}{2}s^2 + 3/2}{s(-\frac{1}{2}s^2 + \frac{1}{2})} = \frac{-\frac{3}{2}(s^2-1)}{-\frac{s}{2}(s^2-1)} = \frac{3}{s} \text{ is PR}$$

note $s^2 - 1 = s^2 - k^2 = (s-k)(s+k)$ cancelled

$$y_1(s) = f(k)\cdot\hat{R}(s) = y(k)\hat{R}(s)$$
$$= \frac{1}{2}\cdot\frac{3}{s} = \frac{3/2}{s} \Rightarrow$$



$$C = f(k)/k = \frac{1/2}{1} = 1/2$$



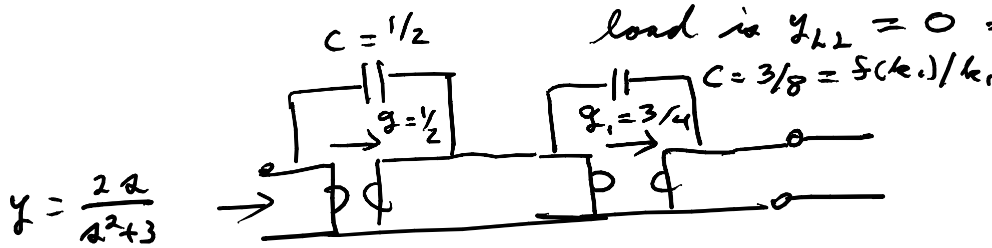
$y_1(s) = \dfrac{3/2}{s}$ $\qquad \hat{R}_1 = \dfrac{k f(k) - s f(s)}{k f(s) - s f(k)}$; $f(s) \Rightarrow y_1(s)$, $k \Rightarrow k_1$

$k_1 = 2$

$y_1(s)\big|_{s=k_1} = 3/4$

$$= \frac{2\cdot 3/4 - s\cdot\frac{3/2}{s}}{2\,\frac{3/2}{s} - s\,3/4} = \frac{3/2 - 3/2}{\frac{3}{s} - 3/4\, s} = 0$$

load is $y_{L2} = 0 \Rightarrow$ open circuit

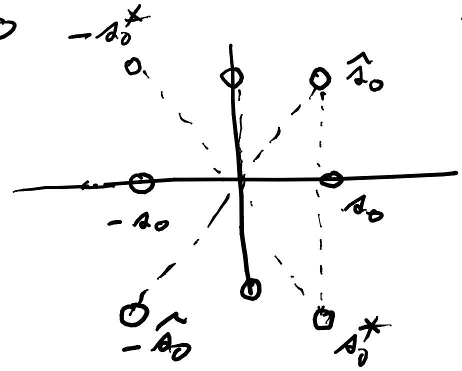


shows by example that any $y(s)$ which is PR and odd (= reactance function) can synthesize by a cascade of gyrator–C circuits using any real $k > 0$.

For any PR $y(s)$ desire to do the same. But we need k to be a zero of $\mathrm{Ev}\, y(s)$ and these need not be real.

$$2\,\mathcal{E}v\, y(s) = y(s) + y(-s) = 0$$

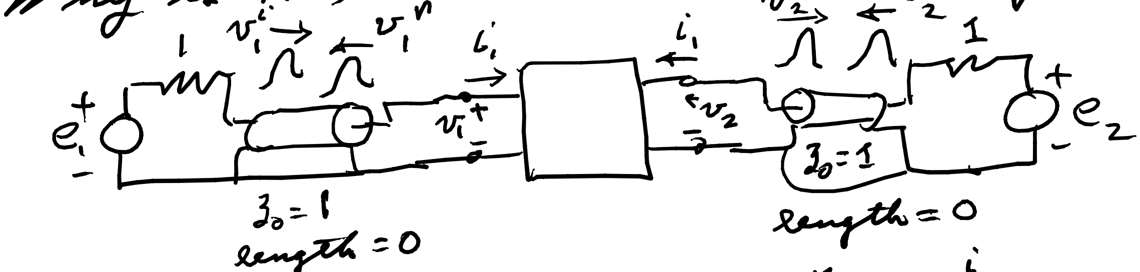
for $y(s)$ rational with real coefficients



s-plane

Can proceed if 1st use k with $\mathrm{Re}\, k > 0$ & then repeat on k^* & combine conjugate components for a PR $y_{LL}(s)$ with $\delta[y_{LL}] = \delta[y] - 2$

Why is $R(s)$ PR: Look at scattering variables



$$2v^i = v + 1\cdot i = v + i \qquad v^r = S v^i$$
$$2v^r = v - 1\cdot i = v - i$$

$$2v = 2v^i + 2v^r \Rightarrow \quad v = v^i + v^r = v^i + S v^i$$
$$2i = 2v^i - 2v^r \Rightarrow \quad i = v^i - v^r = v^i - S v^i$$

$$\text{if } i = Yv = \quad v^i - S v^i = Y(v^i + S v^i)$$
$$= (1_2 - S)v^i = Y(1_2 + S)v^i$$
$$\Rightarrow \quad (1_2 - S) = Y(1_2 + S)$$

$$\text{or} \qquad Y = (1_2 - S)(1_2 + S)^{-1}$$

if a scalar

$$y(s) = \frac{1 - S(s)}{1 + S(s)}$$