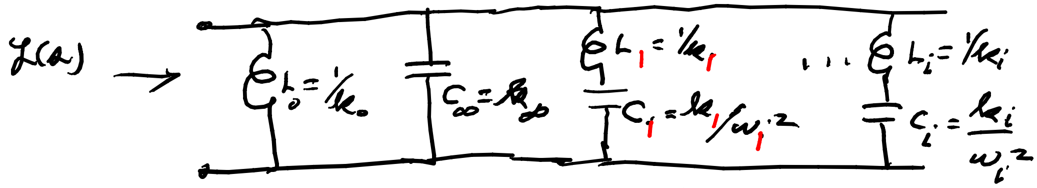
Reactance function synthesis

EE 610 M 10/06/08

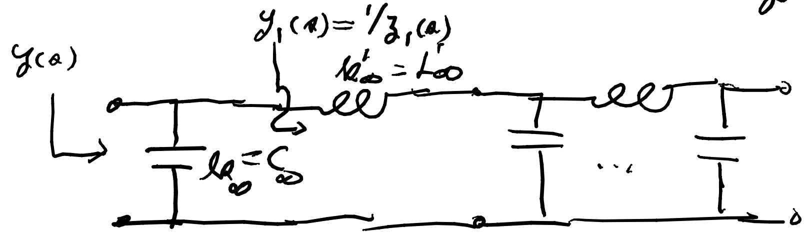
$$y(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{k_i s}{(s^2+\omega_i^2)}$$

$$y_i = \frac{k_i s}{s^2+\omega_i^2}; \quad z_i = \frac{s}{k_i} + \frac{\omega_i^2}{s k_i}$$

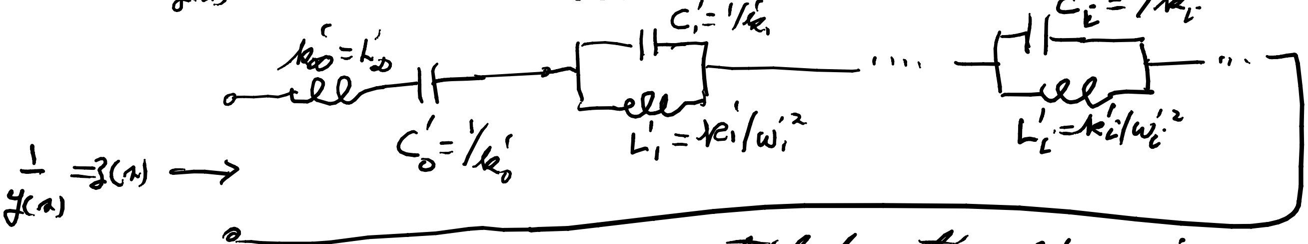


2nd Foster = partial fraction expansion of $y(s) = y(-s)$ reactance

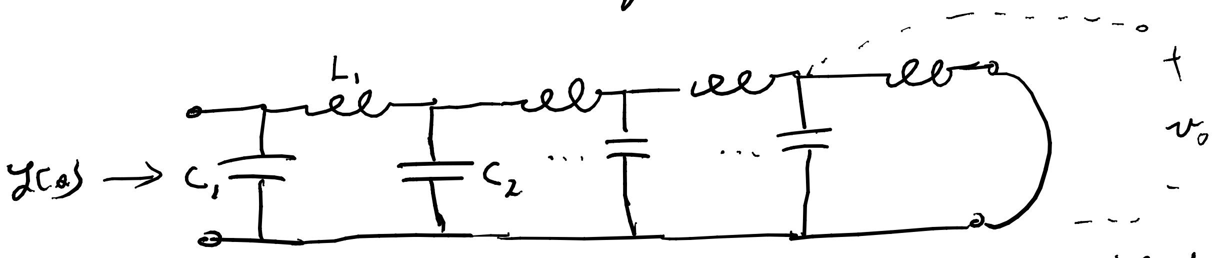
uses minimum number of L's & C's (reactances)



$$z(s) = \frac{1}{y(s)} = k'_\infty s + \frac{k'_0}{s} + \sum_{i=1}^{n'} \frac{k'_i s}{s^2+\omega_i'^2} \qquad \frac{s^2+\omega_i'^2}{k'_i s} = y'_i$$



1st Foster = partial fraction expansion of $z(s) = 1/y(s)$

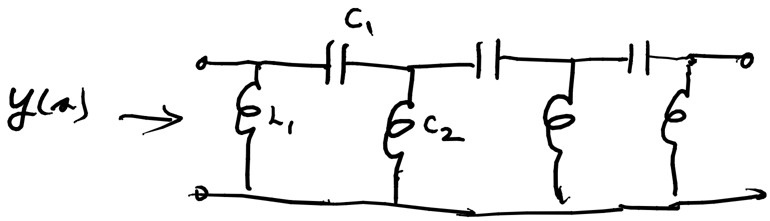


Extract poles at infinity of y's and z's = 1st Cauer

zeros of transmission at ∞

useful for low pass

also can do at $s=0$: extract poles at $s=0$ of y's and z's



2nd Cauer

For 1st Cauer:

$$y(s) = k_\infty s + \cfrac{1}{k'_\infty s + \cfrac{1}{k''_\infty s + \cdots}}$$

Continued fraction expansion about ∞ ($\div$ highest powers of s)

2nd Cauer

$$y(s) = \frac{1}{\frac{1}{k'_0} s} + \cfrac{1}{\cfrac{1}{\frac{1}{k''_0} s} + \cfrac{1}{\cfrac{1}{\frac{1}{k''_0} s} + \cdots}}$$

Continued fraction expansion about $s=0$ ($\div$ lowest powers of s)

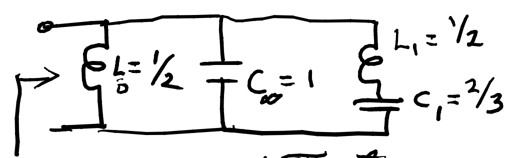
Ex: $$y(s) = \frac{(s^2+1)(s^2+6)}{s(s^2+3)} = \frac{2}{s} + 1\cdot s + \frac{k_1 s}{s^2+3}\,; \quad k_1 = 8/3$$

$$\left.\frac{(s^2+1)(s^2+6)}{s^2}\right|_{s^2=-3} = \left(\underbrace{\frac{2(s^2+3)}{s^2} + (s^2+3)}_{=0} + k_1\right) \qquad \left.\frac{s^2+3}{s}\right|_{s^2=-3}$$

$s^2 = -3$

$$\frac{(-2)(+3)}{-3} = +2 = k_1$$

$$y(s) = \frac{2}{s} + s + \frac{2s}{s^2+3} \Rightarrow$$



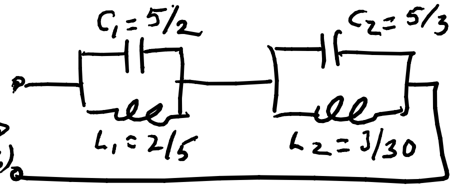
2nd Foster

$$z(s) = \frac{1}{y(s)} = \frac{s(s^2+3)}{(s^2+1)(s^2+6)} = \frac{k_1 s}{s^2+1} + \frac{k_2 s}{s^2+6} = \frac{2s/5}{s^2+1} + \frac{3s/5}{s^2+6}$$

$$k_1 = \frac{s^2+1}{s} \times z(s)\Big|_{s^2=-1} = \left.\frac{\cancel{s}(s^2+3)}{s^2+6}\right|_{s^2=-1} = \frac{\cancel{s}(2)}{+5} \qquad y_1 = \frac{s^2+1}{2s/5}$$

$$k_2 = \frac{s^2+6}{s} \times z\Big|_{s^2=-6} = \frac{-6+3}{-6+1} = \frac{-3}{-5} = 3/5 \qquad y_2 = \frac{s^2+6}{\frac{3}{5}s}$$

$$Z(s) = \frac{s(s^2+3)}{(s^2+1)(s^2+6)} \longrightarrow$$



1st Foster

uses 4 reactive elements

$$\Updownarrow$$

$$y(s) = \frac{(s^2+1)(s^2+6)}{s(s^2+3)} \; ; \quad y(s) = \frac{s^4+7s^2+6}{s^3+3s} = N/D \; ; \quad \delta[y] = 4 = \delta[Z]$$

degree of a rational function, $f \Rightarrow \delta[f]$

1st Cauer: start with a pole at ∞

$$D = s^3+3s \;\big)\; s^4+7s^2+6 = N \quad \text{quotient } s$$

$$s^4 + 3s^2$$

$$4s^2+6 \Rightarrow \text{remainder} = \frac{4s^2+6}{s^3+3s} = \frac{N_1}{D}$$

$$4s^2+6 \;\big)\; s^3+3s \quad \text{quotient } (1/4)s$$

$$s^3+\frac{6}{4}s$$

$$\frac{12-6}{4} = \frac{6}{4}s \Rightarrow \frac{\frac{6}{4}s}{4s^2+6}$$

$$\frac{6}{4}s \;\big)\; 4s^2+6 \quad \text{quotient } \frac{16}{6}s$$

$$4s^2$$

$$6 \Rightarrow \frac{6}{6/4\, s}$$

$$6 \;\big)\; 6/4\, s \quad \text{quotient } \frac{36}{4}s$$

$$6/4\, s$$

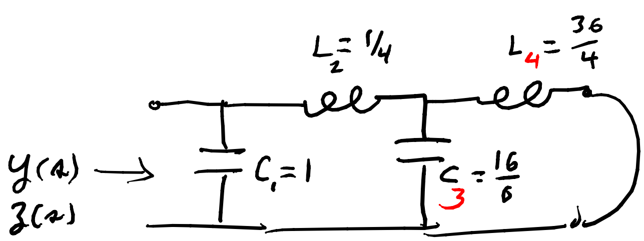
$$0$$

$$y(s) = s + \cfrac{1}{\frac{1}{4}s + \cfrac{1}{\frac{16}{6}s + \cfrac{1}{\frac{36}{4}s + 0}}}$$

y, Z, y, Z

$\Downarrow$

$Z = 0 =$ short



1st Cauer

2nd Cauer $\quad y(s) = \dfrac{s^4+7s^2+6}{s^3+3s}$; $Z(s)$

$$3s+s^3 \;\big)\; 6+7s^2+s^4 \quad \text{quotient } \frac{6}{3s}$$

$$6+\frac{6}{3}s^2$$

$$\frac{15}{3}s^2 + s^4$$

$$\frac{15}{3}s^2+s^4 \;\big)\; 3s+s^3 \quad \text{quotient } \frac{9}{15s}$$

$$3s \quad \frac{9}{15}s^3$$

$$\frac{6}{15}s^3 \;\big)\; \frac{15}{3}s^2 + s^4 \quad \text{quotient } \frac{15^2}{3\times 6 s}$$

$$\frac{15}{3}s^2$$

$$s^4 \;\big)\; \frac{16}{15}s^3 \quad \text{quotient } \frac{16}{15s}$$

$$Y(s) = \underbrace{\frac{6}{3s}}_{y} + \cfrac{1}{\underbrace{\frac{9}{15s}}_{z} + \cfrac{1}{\underbrace{\frac{15^2}{18s}}_{y} + \cfrac{1}{\underbrace{\frac{16}{15s}}_{z} + 0}}} \Rightarrow$$