Synthesis of PR functions EE 610 W 10/01/08

PR = rational positive real $f(s)$

$f(\cdot)$ an admittance or an impedance

$$\underbrace{\quad y(s) \qquad\qquad\qquad\qquad z(s) = 1/y(s)\quad}_{y \text{ or } z \Rightarrow \text{immittance}}$$

$$\left.\begin{aligned} V(s) &= z(s)I(s) \\ I(s) &= y(s)V(s) \end{aligned}\right\}$$ duals if replace I by V & V by I

in the dual replace

$C_d = L$, $L_d = C$, $R_d = R$

then $y_d(s) = z(s) = 1/y(s)$

know if $f(s)$ is PR then $1/f(s)$ is PR

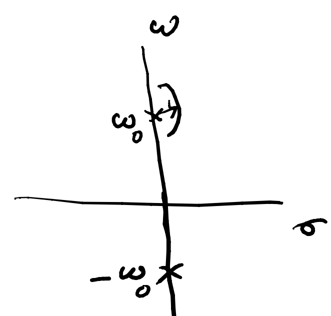
then $f(1/s)$ is PR

for PR $$y(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + \cdots + a_1 s + a_0}{b_m s^m + \cdots + b_1 s + b_0}$$

has no poles in RH s-plane ; all in $s \le 0$

no zeros in RH s-plane ; all in $s \le 0$

$$y(s) = \frac{k_0}{(s - j\omega_0)^{m_0}} + \cdots$$ $k_0 > 0$; $m_0 = 1$



as coefficients are real any zero of N or D comes with a conjugate

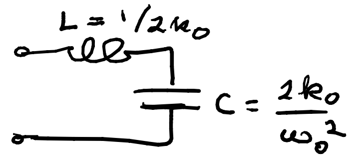
$$y(s) = \frac{k_0}{s - j\omega_0} + \frac{k_0}{s + j\omega_0} + \text{others}$$ terms in a partial fraction expansion

$$= \frac{k_0(s + j\omega_0) + k_0(s - j\omega_0)}{(s - j\omega_0)(s + j\omega_0)} +$$

$$= \frac{2k_0 s}{s^2 + \omega_0^2} + y_1(s)$$; set $$y_0(s) = \frac{2k_0 s}{s^2 + \omega_0^2}$$

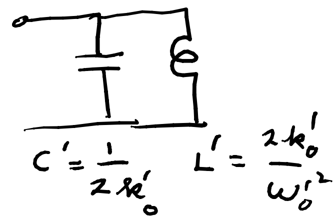
$$z_0(s) = \frac{1}{y_0(s)} = \frac{s}{2k_0} + \frac{\omega_0^2}{2k_0 s}$$

$y_0 = \frac{1}{z_0} \longrightarrow$



If $z(s) = \dfrac{2k_0' s}{s^2+\omega_0'^2} + z_1(s)$

$z_0' = \dfrac{1}{y_0'} = \dfrac{2k_0' s}{s^2+\omega_0'^2} \longrightarrow$



For a reactance function

$y(s) + y(-s) = 0 \Rightarrow$ no poles of $y(s)$ in $\sigma \neq 0$ (none in RHP or LHP) or zeros

$\dfrac{1}{z(s)} + \dfrac{1}{z(-s)} = 0$

$\dfrac{z(s)+z(-s)}{z(s)z(-s)} = 0 \Rightarrow$ no poles of $z(s)$ in $\sigma \neq 0$

but poles of $z(s) = \dfrac{1}{y(s)} = \dfrac{1}{N/D} = \dfrac{D}{N}$ in $\sigma \neq 0$

$\therefore$ for a reactance function all poles and zeros are on the $j\omega$ axis & there the poles (and zeros) are simple with a positive residue.

$$y(s) = \frac{\prod_{i=1} (s^2 + \omega_{n_i}^2)}{\prod_j (s^2 + \omega_{d_j}^2)} \times \frac{1}{s} \quad (\text{or} \times s)$$

Ex: $y(s) = \dfrac{1}{s}\dfrac{(s^2+3)}{(s^2+4)}$ maybe $\hat{y}(s) = \dfrac{s^2+3}{s(s^2+2)}$ but not a reactance function:

$$= \frac{k_0}{s} + \frac{k_1}{s-j2} + \frac{k_1^*}{s+j2}$$

k_0: $s\,y(s)\Big|_{s=0} = k_0 + s\left[\dfrac{k_1}{s-j2} + \dfrac{k_1^*}{s+j2}\right]\Bigg|_{s=0} = k_0 \Rightarrow k_0 = 3/4$

$$= s\,\frac{1}{s}\left(\frac{s^2+3}{s^2+4}\right)\Bigg|_{s=0} = 3/4$$

k_1: $(s-j2)\,y(s)\Big|_{s=j2} = \dfrac{k_0}{s}(s-j2)\Big|_{s=j2} + k_1 + \left(\dfrac{s-j2}{s+j2}\right)k_1^*\Big|_{s=j2} = k_1$

$$\left.(s-j2)\frac{1}{s}\left(\frac{s^2+3}{(s-j2)(s+j2)}\right)\right|_{s=j2} = \frac{-4+3}{j2(2\times j2)} = \frac{-1}{-8} = 1/8 = k_1 = k_1^*$$

$$y(s) = \frac{1}{s}\frac{(s^2+3)}{s^2+4} = \frac{3/4}{s} + \frac{1/8}{s-j2} + \frac{1/8}{s+j2} = \frac{3/4}{s} + \frac{2\times\frac{1}{8}s}{s^2+4}$$

as a check $\Rightarrow$

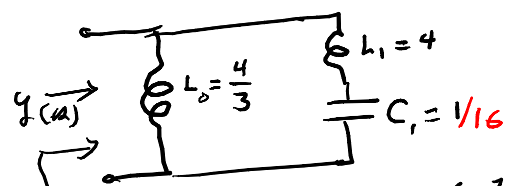
$$\frac{\frac{3}{4}(s^2+4)+\frac{2}{8}s^2}{s(s^2+4)} = \frac{1.\,s^2+\frac{3\times4}{4}}{s(s^2+4)}$$

here

$$y(s) = \frac{3/4}{s} + \frac{\frac{1}{4}s}{s^2+4}$$

$$/\!/ \text{ th}$$

$$y_1 \to z_1 = \frac{1}{y_1} = \frac{s^2+4}{\frac{1}{4}s} = 4s + \frac{1}{\frac{1}{16}s}$$



$$z(s) = \frac{1}{y(s)} = \frac{s(s^2+4)}{s^2+3}$$

Look at $\hat{y} = \frac{1}{s}\cdot\frac{s^2+3}{s^2+2} = \frac{3/2}{s} + \frac{\hat{k}_1}{s-j\sqrt{2}} + \frac{\hat{k}_1^*}{s+j\sqrt{2})}$

$$\hat{k}_1 = \left.(s-j\sqrt{2})\hat{y}\right|_{s-j\sqrt{2}=0} = \hat{k}_1$$

$$= \left.\frac{1}{j\sqrt{2}}\cdot\frac{3-2}{(s+j\sqrt{2})}\right|_{s=j\sqrt{2}} = \frac{1}{j\sqrt{2}\cdot 2\cdot j\sqrt{2}} = -1/4$$

$\therefore$ $\hat{y}(s)$ is not PR

for a reactance function (is PR, $y(s)+y(-s)=0$)

$$y(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{q}\frac{k_i s}{s^2+\omega_i^2}$$

all k's real & positive

$$y(j\omega) = j\left[-\frac{k_0}{\omega} + k_\infty\omega + \sum_{i=1}^{q}\frac{k_i\omega}{-\omega^2+\omega_i^2}\right]$$

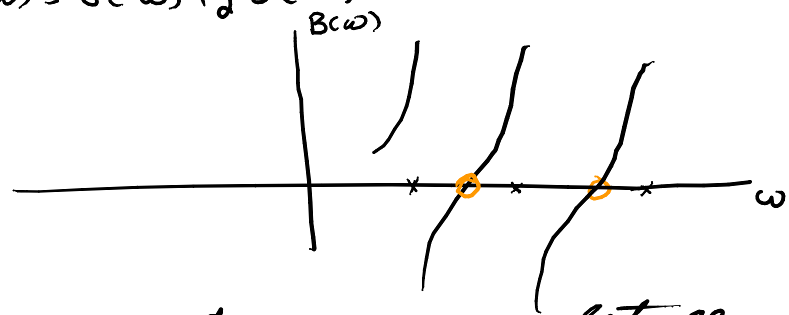
$$\frac{d\,y(j\omega)/j}{d\omega} = \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{q}\left\{\frac{k_i}{-\omega^2+\omega_i^2} - \frac{k_i\omega(-2\omega)}{(-\omega^2+\omega_i^2)^2}\right\}$$

$d\,1/x = -\frac{1}{x^2}$

$$= \frac{k_0}{\omega^2} + k_\infty + \sum_{i=1}^{q}\frac{+1k_i\omega^2+k_i\omega_i^2}{(-\omega^2+\omega_i^2)^2}$$

we see $\frac{d\, Y(j\omega)/j}{d\omega} > 0$ (or ∞ at the poles)

$Y(j\omega) = G(\omega) + j\,B(\omega)$



shows that zeros appear between poles of a reactance function $\Rightarrow$ poles & zeros alternate

note $\hat{Y} = \frac{s^2+3}{s(s^2+2)}$ does not have this property