Passive Synthesis = Chapter 8

Given a transfer function to create a circuit,

Start with 1-port $y(s)$ or $z(s)$

assume from a passive finite RLC (gyrators & transformers) circuit

then $y(s)$ & $z(s)$ are rational in s

$$y(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0}{b_m s^m + \dots b_1 s + b_0}$$

has real coefficients, no poles in the RHP, $s = \sigma + j\omega$, $\sigma > 0$

also require by the passiveness that

$Re\, y(s) \geq 0$ in $\sigma > 0$

Positive-Real function $f(s)$

1) $f(s)$ is real for $s = \sigma$ (s=real) with $\sigma > 0$

[means real components a_i & b_i all real if rational]

2) $f(s)$ is analytic in $s = \sigma > 0$

[means, stable circuit; no poles in $\sigma > 0$ if rational]

3) $Re\, f(s) \geq 0$ in $s = \sigma + j\omega$ with $\sigma > 0$

[means $f(s)$ comes from a passive circuit]

Ex:

1) $f(s) = y(s) = g \implies g$ real — for 1)

for 2) automatic

$Re\, g = g \geq 0$ — for 3)

2) $f(s) = \frac{1}{Ls} = y(s)$ 

$f(\sigma) = \frac{1}{L\sigma}$ should be real for σ real $> 0 \Rightarrow L = \text{real}$

$f(s) = \frac{1}{Ls}$ has a pole at $s = 0$ not in $\sigma > 0$

2) $\Rightarrow$ ok

$$Re\, f(s) = Re\left(\frac{1}{L(\sigma + j\omega)}\right) = \frac{1}{2}\left[\frac{1}{L(\sigma+j\omega)} + \frac{1}{L(\sigma - j\omega)}\right]$$

$$= \frac{1}{2}\left[\frac{\sigma - \cancel{j\omega} + \sigma + \cancel{j\omega}}{L(\sigma+j\omega)(\sigma-j\omega)}\right] = \frac{\sigma}{L(\sigma^2+\omega^2)}$$

in $\sigma > 0$ for all ω, $-\infty < \omega < \infty$

$\Rightarrow L > 0 \Rightarrow$ shows  is passive

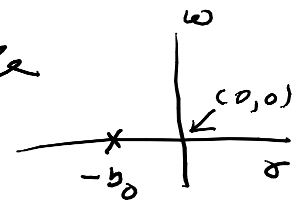
3) $f(s) = \frac{s+2}{s+b_0}$ & find b_0 such that $f(s)$ is PR

condition 1) $b_0 \Rightarrow$ real

condition 2) $s = -b_0$ is the only pole

can't be in $\sigma > 0$

$\Rightarrow b_0 \geqslant 0$



condition 3)

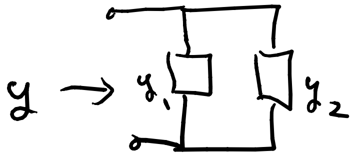
$$Re\, f(s) = \frac{1}{2}\left\{\frac{\sigma + j\omega + 2}{\sigma + j\omega + b_0} + \frac{\sigma - j\omega + 2}{\sigma - j\omega + b_0}\right\}$$

$$= \frac{1}{2}\left\{\frac{(\sigma + 2 + j\omega)(\sigma - j\omega + b_0) + (\sigma + 2 - j\omega)(\sigma + b_0 + j\omega)}{(\sigma + b_0 + j\omega)(\sigma + b_0 - j\omega)}\right\}$$

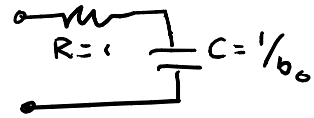
$$= \frac{1}{2}\left\{\frac{2\left((\sigma+2)(\sigma+b_0) + \omega^2\right)}{(\sigma+b_0)^2 + \omega^2}\right\} \geqslant 0$$ for all $\sigma > 0$ & all $-\infty < \omega < \infty$

$y(s)\ f(s) = \frac{s+2}{s+b_0}$ is PR for $b_0 \geqslant 0$ & of degree $f(s) = 1$

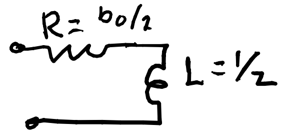
$$= \frac{s}{s+b_0} + \frac{2}{s+b_0} = \frac{1}{1+\frac{b_0}{s}} + \frac{1}{\frac{s}{2} + \frac{b_0}{2}} = y(s) = y_1 + y_2$$



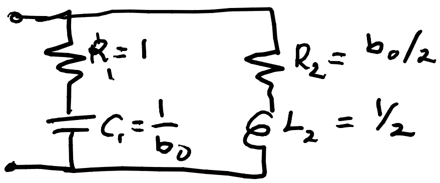
$y_1 \Rightarrow z_1 = \frac{1}{y_1} = 1 + \frac{b_0}{s} \Rightarrow$



$y_2 \Rightarrow z_2 = \frac{1}{y_2} = \frac{b_0}{2} + \frac{s}{2} \Rightarrow$



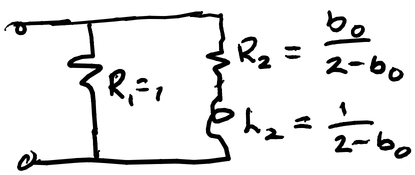
$y(s) = \frac{s+2}{s+b_0} \Rightarrow$



Rewrite $f(s) = \frac{s+2}{s+b_0} = y(s)$

$$\begin{array}{r} 1 \\ s+b_0 \overline{\smash{)}\, s+2} \\ \underline{s+b_0} \\ 2-b_0 \end{array}$$

$= 1 + \frac{2-b_0}{s+b_0}$



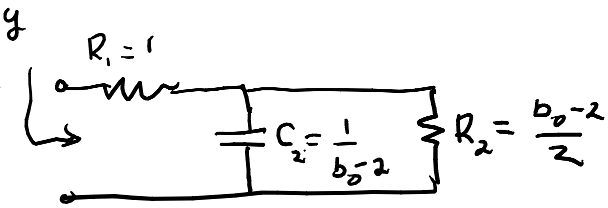
this passive only if $0 < b_0 < 2$

$y(s) = f(s) = \cfrac{1}{\cfrac{s+b_0}{s+2}}$

$$\begin{array}{r} 1 \\ s+2 \overline{\smash{)}\, s+b_0} \\ \underline{s+2} \\ b_0-2 \end{array}$$

$= \cfrac{1}{\underbrace{1}_{z_1} + \underbrace{\cfrac{b_0-2}{s+2}}_{z_2}} \Rightarrow$

$y_2 = \frac{1}{z_2} = \frac{s+2}{b_0-2}$



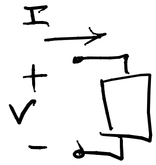
valid for $b_0 > 2$

shows can have many solutions to getting a circuit from a transfer function.

circuits without resistance $\Rightarrow$ lossless

$\Rightarrow$ reactance functions, p. 342

$P(j\omega) = 0 =$ average power input in the sinusoidal steady state for a lossless circuit



$$P(j\omega) = \frac{V^* I + V I^*}{2} \quad \text{at } s = j\omega$$

$$= \frac{V^* y(j\omega) V + V y^*(j\omega) V^*}{2}$$

$$= \frac{1}{2}\left\{ y(j\omega) V^* V + y^*(j\omega) V V^* \right\}$$

$$= \frac{1}{2}\left(y(j\omega) + y^*(j\omega) \right) V V^* = 0 \text{ when } V \neq 0$$

$$y(j\omega) + y^*(j\omega) = 0$$

$$= y(j\omega) + y(-j\omega) = 0$$

$$= \left[y(s) + y(-s) \right] \Big|_{s=j\omega} = 0$$

for almost all ω (not really for natural frequencies [a finite # of them])

mathematicians show this will hold for all s

$$y(s) + y(-s) = 0 \implies y(s) = -y(-s)$$

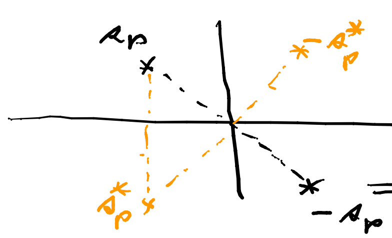
$$\shortparallel$$

$$2\, \mathcal{E}v\, y(s) = 0$$

Ex: $y(s) = \dfrac{1}{Ls}$ = $\quad y(s) + y(-s) = \dfrac{1}{Ls} + \dfrac{1}{L(-s)} = \dfrac{1}{Ls} - \dfrac{1}{Ls} \equiv 0$

$$\left.\begin{aligned} \mathcal{E}v\, y(s) &= \tfrac{1}{2}\left[y(s) + y(-s) \right] \\ \mathcal{O}d\, y(s) &= \tfrac{1}{2}\left[y(s) - y(-s) \right] \end{aligned}\right\} \quad y(s) = \mathcal{E}v\, y(s) + \mathcal{O}d\, y(s)$$

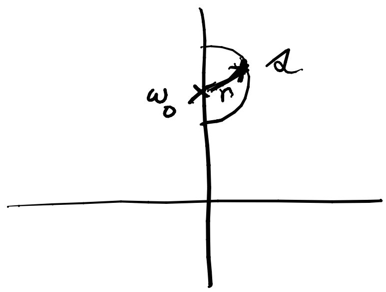
$\implies y(s) = \mathcal{O}d\, y(s)$ for a reactance function



$\implies$ can not be if $y(s)$ is PR

ii for a reactance function all poles are on the $j\omega$ axis $\frac{k}{(s+j\omega_0)^{m_p}}$

but these poles must be simple ($m_p = 1$ & k real) with real positive residue ($k > 0$)



$Y(s)$ near the pole

$$\approx \frac{k}{(s-j\omega_0)^{m_0}} \quad \text{for} \quad re^{j\theta}, \; -\frac{\pi}{2} < \theta < \pi/2$$

$$(s - j\omega_0) = re^{j\theta}$$

here $\operatorname{Re} Y(s) \geq 0$;

$$\operatorname{Re} Y(s)\Big|_{\text{on the semicircle}} = \operatorname{Re}\left(\frac{k}{(re^{j\theta})^{m_0}}\right) = \frac{1}{r}\operatorname{Re}\left(\frac{k}{e^{j\theta m_0}}\right)$$

$$= \frac{1}{r}|k|\,\operatorname{Re}\left(e^{j(\angle k - \theta m_0)}\right)$$

$$= \frac{1}{r}|k|\cos(\angle k - \theta m_0) > 0 \text{ only if}$$

$\angle k = 0$

$m_0 = 1$