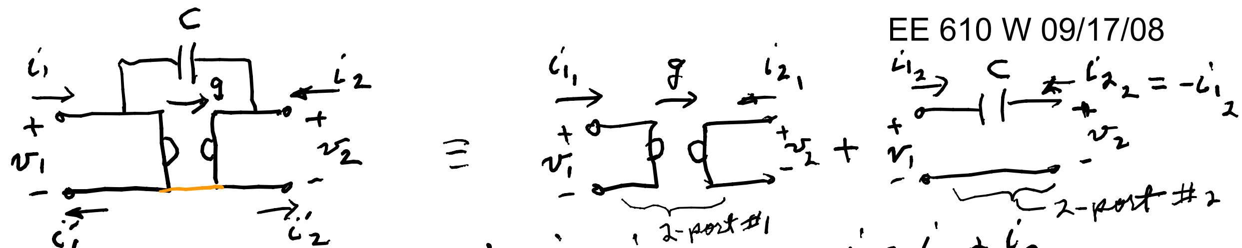
EE 610 W 09/17/08



$i = Yv$

$i_1 = i_{1_1} + i_{1_2}$ $\qquad$ $i_2 = i_{2_1} + i_{2_2}$

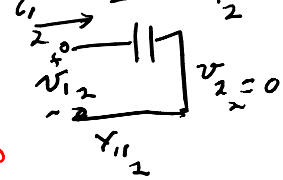
$v_{1_1} = v_{1_2} = v_1$ $\qquad$ $v_{2_1} = v_{2_2} = v_2$

$$i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Y v = \begin{bmatrix} i_{1_1} + i_{1_2} \\ i_{2_1} + i_{2_2} \end{bmatrix}$$

$$= Y_1 v + Y_2 v = (Y_1 + Y_2) v$$

$y_{11} = \frac{i_1}{v_1}\Big|_{v_2 = 0}$

$i_{1_2} = Cs\, v_{1_2}$

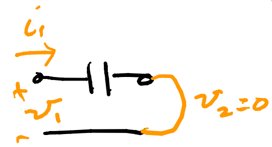


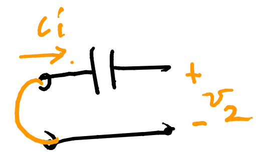
$y_{12} = i_{1_2} / v_{2_2} \Big|_{v_{1_2}}$

$$Y_1 = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \; ; \qquad Y_2 = \begin{bmatrix} Cs & -sC \\ -Cs & sC \end{bmatrix}$$

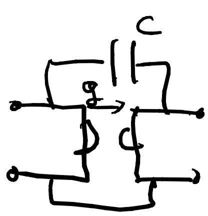
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow y_{11} = i_1/v_1 \Big|_{v_2=0}$$

$$y_{12} = i_1/v_2 \Big|_{v_1=0}$$





$= -sC$



$$\Rightarrow Y = Y_1 + Y_2 = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}$$

$$= \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}$$

$$Z = Y^{-1} = \frac{1}{\det Y} \begin{bmatrix} \overset{=(1,1)\text{ cofactor}}{sC} & \overset{=(2,1)\text{ cofactor}}{-g + sC = -(g - sC)} \\ g + sC & sC \end{bmatrix}$$

$$\det = (sC)(sC) - (g - sC)(-g - sC) = \cancel{s^2C^2} - (-g^2 + \cancel{s^2C^2})$$

$$= +g^2$$

$$Z = \frac{1}{g^2} \begin{bmatrix} sC & -g + sC \\ g + sC & sC \end{bmatrix}$$ as a check $Z \cdot Y = 1_2$

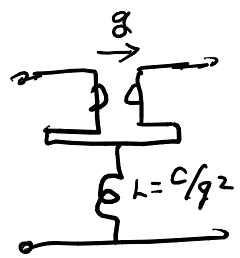
$$ZY = \frac{1}{g^2}\begin{bmatrix} ac & -g+ac \\ g+ac & ac \end{bmatrix}\begin{bmatrix} ac & g-ac \\ -g-ac & ac \end{bmatrix} = \frac{1}{g^2}\begin{bmatrix} a^2c^2+g^2-a^2c^2 & gac-a^2c^2-gac+a^2c^2 \\ gac+a^2c^2-gac-a^2c^2 & g^2-a^2c^2+a^2c^2 \end{bmatrix}$$

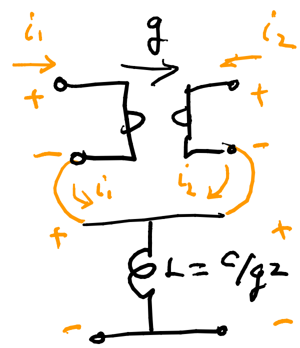
$$= \frac{1}{g^2}\begin{bmatrix} g^2 & 0 \\ 0 & g^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} ac/g^2 & -\frac{1}{g} + ac/g^2 \\ \frac{1}{g} + \frac{ac}{g^2} & a\frac{c}{g^2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1/g \\ 1/g & 0 \end{bmatrix} + \frac{ac}{g^2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

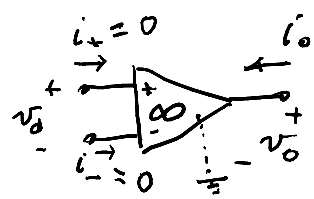
$V = ZI$

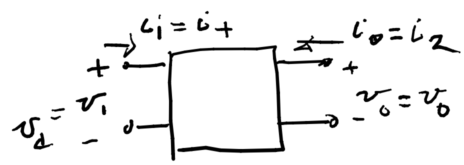




Some things have no admittance or impedance descriptions

ideal op-amps:



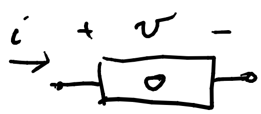


$v_{in} = v_d = 0$; $i_+ = i_- = 0$

v_0 = arbitrary, i_0 = arbitrary

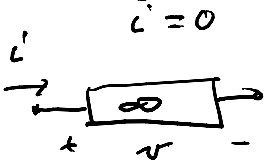
virtual "ground"

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{\text{ideal op-amp}}$$

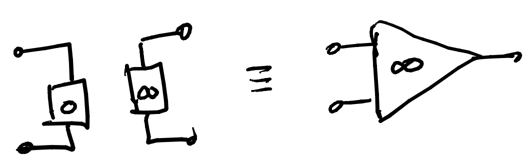


nullator

$v = 0$
$i = 0$



norator



the nullor 2-port

$$[0][i] = [0][v] \Rightarrow \text{norator}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}[i] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}[v] \Rightarrow \text{nullator}$$

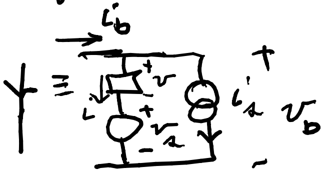
for linear circuit $\quad Av = Bi \quad$ as port descriptions

For Z: $\quad v = Zi$, $\quad Z = A^{-1}B$

Y: $\quad i = Yv \quad Y = B^{-1}A$

no Z or Y for an ideal op-amp

Back to graph theory: $\quad Av = Bi$



$$i_b = i + i_s = i + j$$
$$v_b = v + v_s = v + e$$

solve for $i, v \Rightarrow \quad Av = A(v_b - v_s) = B(i_b - i_s)$

$i_b = \mathcal{J}^T \cdot i_\ell \qquad A v_b - B i_b = \underbrace{A v_s - B i_s}_{\text{source terms}}$

$v_b = e^T \cdot v_t \qquad A e^T v_t - B \mathcal{J}^T i_\ell =$

$$= [Ae^T \;\; -B\mathcal{J}^T] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = A v_s - B i_s$$

$$x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$$

$$= [Ms + N]\, x = \mathcal{B} \cdot u \qquad u = \text{input sources}$$

$$\underset{\underset{E}{\parallel}}{M} s \cdot x = \underbrace{-N}_{A} x + Bu \Rightarrow E \frac{dx}{dt} = Ax + \mathcal{B}u$$

$$x = \text{semi-state} = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$$

If output is a linear combination of voltages and currents then it is given by a matrix on x (tree voltages & links currents)

$$y = C \cdot x \qquad y = \text{output}$$

or

$$\left.\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\}$$

E can be singular
u = inputs
y = outputs
$x = x(t)$ = semi-state

If E is identity

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\} \text{state variable equations}$$

$$+ Du + F\dot{u}$$