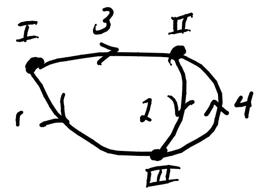
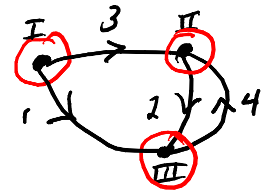
Incidence matrix
augmented

for each node list the oriented branches going out of (or into) the node

A_a is $n \times b$ matrix of $0, \pm 1$'s.

Ex:





$$A_a = \begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \uparrow \text{nodes} \end{array} \begin{array}{c} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 0 & 1 \end{bmatrix} \\ \begin{matrix} 1 & \;\; 2 & \;\; 3 & \;\; 4 \end{matrix} \leftarrow \text{branches} \end{array}$$

rank

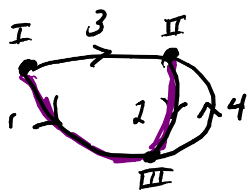
$$A_a A_a^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -2 \\ -1 & -2 & 3 \end{bmatrix}$$

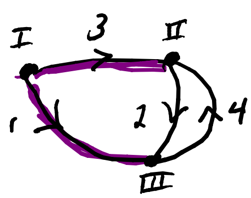
$$\det(A_a A_a^T) = 18 + (-2) + (-2) - 3 - 8 - 3$$
$$= 14 - 14 = 0$$

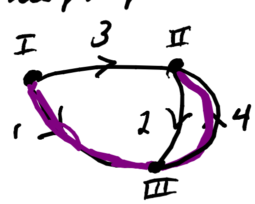
remove 3rd row

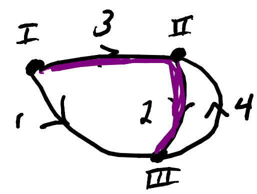
$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}; \quad AA^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

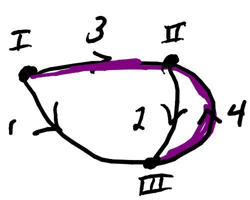
$$\det(AA^T) = 6 - 1 = 5 = \text{\# of trees in the graph}$$



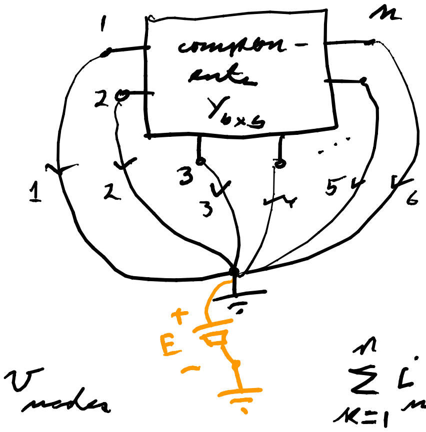








Indefinite admittance



$$i_{node} = -Y_{ind} \cdot v_{nodes}$$

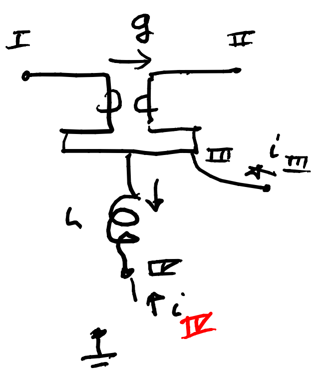
$$\sum_{k=1}^{n} i_{node_k} = 0 \Rightarrow$$ entries in a column of Y_{ind} sum to 0

$$= -Y_{ind} \cdot \begin{bmatrix} v_{node_1} + E \\ \vdots \\ v_{node_n} + E \end{bmatrix} = -Y_{ind} \cdot v_{node} - \underbrace{Y_{ind} \cdot E \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{=0}$$

for any E

$= 0 \Rightarrow$ sum of entries in any row $= 0$

Ex:



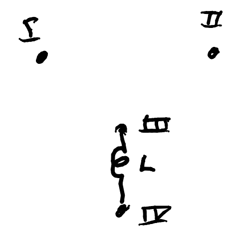
Y_{ind} is 4×4

for the inductor

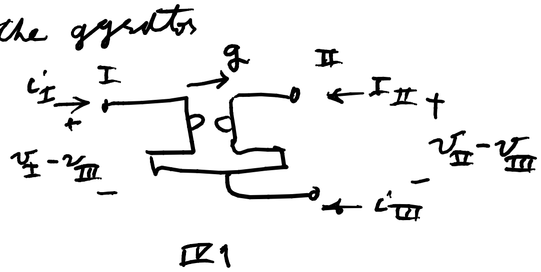
$$i_{ind} = \frac{1}{sL} \cdot (v_{III} - v_{IV}) = i_{III} \text{ - gyrator current}$$

$$i_{IV} = -\left(\frac{1}{sL}\right)(v_{III} - v_{IV})$$

$$Y_{ind \text{ for } L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{sL} & -\frac{1}{sL} \\ 0 & 0 & -\frac{1}{sL} & \frac{1}{sL} \end{bmatrix} \Rightarrow$$



for the gyrator



$$\begin{bmatrix} i_I \\ i_{II} \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_I - v_{III} \\ v_{II} - v_{III} \end{bmatrix}$$

$$i_{III} + i_I + i_{II} = 0$$

$$i_{III} = -i_I - i_{II}$$

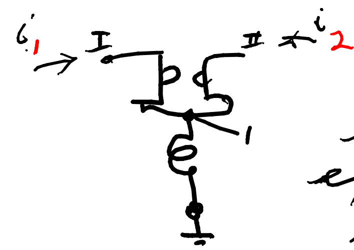
$$Y_{ind\,gyr} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ +g & -g & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{I} \\ v_{II} \\ v_{III} \\ v_{IV} \end{bmatrix}$$

$$Y_{ind} = Y_{ind\,gyr} + Y_{ind\,L} = \begin{bmatrix} 0 & g & -g & 0 \\ -g & 0 & g & 0 \\ +g & -g & +\frac{1}{sL} & -\frac{1}{sL} \\ 0 & 0 & -1/sL & \frac{1}{sL} \end{bmatrix}$$

gives currents into all the nodes with a ground outside

If move the ground onto the circuit it sets the voltage at the node it is at to zero.
∴ can scratch out that column. Ex. set ground at node 4: (can scratch out the corresponding row)

$$Y_{node} = \begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & \frac{1}{sL} \end{bmatrix}$$



if don't excite from the outside the current in will be zero

If $i_{III} = 0$ then

$$\begin{bmatrix} i_1 \\ i_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & g & -g \\ -g & 0 & g \\ g & -g & 1/sL \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_{III} \end{bmatrix} \Rightarrow \begin{bmatrix} i \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v \\ v_{internal} \end{bmatrix}$$

$$0 = \begin{bmatrix} g & -g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{sL} \end{bmatrix} v_{III} \Rightarrow v_{III} = -\left(\frac{1}{sL}\right)^{-1} \begin{bmatrix} g & -g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= -Y_{22}^{-1} \cdot Y_{21} \cdot v$$

internal node voltages

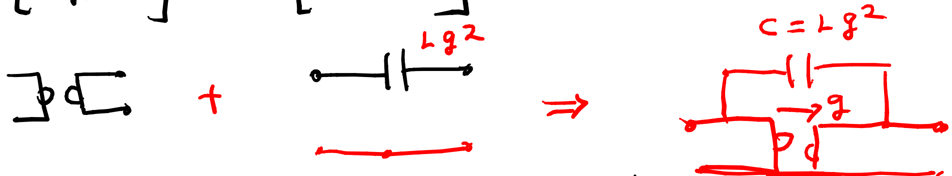
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = i = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} v + \begin{bmatrix} -g \\ g \end{bmatrix} \left(-\frac{1}{sL}\right)^{-1} \begin{bmatrix} g & -g \end{bmatrix} v$$

$$i = Y_{11} v + Y_{12}(-Y_{22}^{-1}) Y_{21} \cdot v$$

$$= \begin{bmatrix} g^2/sL^{-1} & g - g^2/sL^{-1} \\ -g - \frac{g^2}{sL^{-1}} & g^2/sL^{-1} \end{bmatrix} v$$

$$Y_{2port} = \begin{bmatrix} g^2/sL^{-1} & g - g^2/sL^{-1} \\ -g - g^2/sL^{-1} & g^2/sL^{-1} \end{bmatrix}, \quad Y = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + g^2 sL \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



$$Y(s) = \begin{bmatrix} sLg^2 & g - sLg^2 \\ -g - sLg^2 & sLg^2 \end{bmatrix}$$