$v_b = e^T v_t$ , $i_b = \mathcal{J}^T i_\ell$

EE 610 W 09/10/08

$b = t + \ell$

$P = 0 = v_b^T i_b$ = total power from outside the circuit

$= (e^T v_t)^T i_b = (v_t^T e^{T^T}) \cdot i_b = v_t^T \cdot \underbrace{e \cdot i_b}_{\Downarrow \, 0 \text{ as } v_t \text{ is arbitrary}} = 0$

(for the graph with any elements giving it)

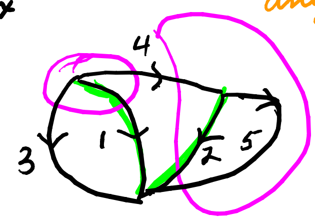
$0 = e i_b$ ; $e$ is $t$ rows, $b$ columns $\} = t \times b$ matrix

$\Downarrow$

$0_t \Rightarrow$ KCL for each tree branch

$v_b = [v_1, v_2, v_3, v_4, v_5]_b^T$

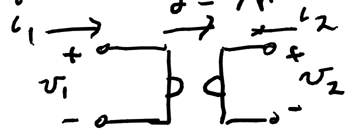
$v_t = [v_1, v_2]_b^T$




$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b ; \quad e = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

gyrator $\Rightarrow$ 2-port $\quad g = 1/r$

$r$ = gyration resistance




$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \Rightarrow v = z\, i$$

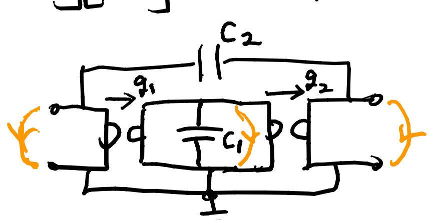
$$z = -z^T = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}$$

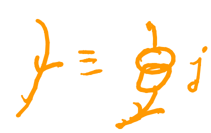
Telegen introduced @ Philips labs

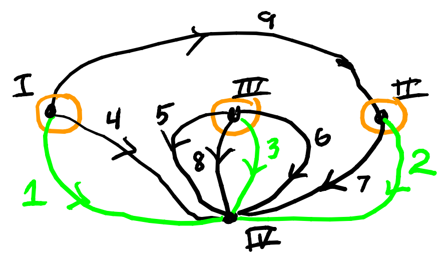
$$z^{-1} = Y = \frac{1}{r^2}\begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$zY = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} = \begin{bmatrix} -r\cdot(-g) = 1 & 0 \\ 0 \cdot r - 0 \cdot g = 0 & r \cdot g = 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{array}{l} i_2 = -g v_1 \\ i_1 = g \cdot v_2 \end{array} \Rightarrow \text{VCCS } (= \text{OTA})$$



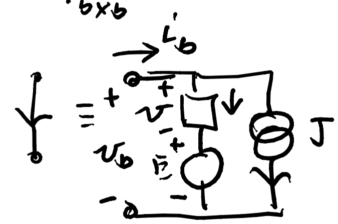



$e$ is $3\times 9$
KCL

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{array}{cccccccccc} & & & 4 & 5 & 6 & 7 & 8 & 9 \\ \end{array}$$

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \left[\begin{array}{ccc|cccccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6\\ i_7\\ i_8\\ i_9\end{bmatrix}_b \; ; \quad v_b = e^T \cdot v_t$$

$$\underset{(9\times 9)}{Y_{b\times b}} = \left[\begin{array}{c|cc|c|c} \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & & 0 & & \\ \hline & \begin{matrix}0 & g_1\\ -g_1 & 0\end{matrix} & & 0 & \\ \hline 0 & & \begin{matrix}0 & g_2\\ -g_2 & 0\end{matrix} & & 0 \\ \hline & 0 & 0 & sC_1 & 0 \\ & & & 0 & sC_2 \end{array}\right] \Rightarrow \quad i = Y_{b\times b} \cdot v$$




$$\left.\begin{aligned} v_b &= v + E \\ i_b &= i + J \end{aligned}\right\} \quad \begin{aligned} v &= v_b - E \\ i &= i_b - J \end{aligned}$$

*here in this example*
$E = 0_9$ = 9-vector of zeros

$$J = \begin{bmatrix} j_1\\ j_2\\ j_3\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

$$i = Y_{b\times b}\, v = i_b - J$$
$$= Y_{b\times b}\, v_b = i_b - J$$

*multiply by* $e \Rightarrow$

$$e\, Y_{b\times b}\, v_b = e \cdot i_b - e \cdot J = 0_3 - eJ = -\begin{bmatrix} j_1\\ j_2\\ j_3\end{bmatrix}$$

$$\|$$

$$e\, Y_{b\times b}\, e^T v_t = -\begin{bmatrix} j_1\\ j_2\\ j_3\end{bmatrix} \Rightarrow \quad Y_{3\times 3} = e\, Y_{b\times b}\, e^T = Y_{nodal}$$

$$v_t = Y_{nodal}^{-1} \cdot i_{\text{into nodes}} = Y_{nodal}^{-1} \cdot (-j) \; ; \quad v_b = e^T v_t$$

$$
\left[\begin{array}{ccc|cccccc}
 & & & 4 & 5 & 6 & 7 & 8 & 9\\
1&0&0&1&0&0&0&0&1\\
0&1&0&0&0&0&1&0&-1\\
0&0&1&0&1&1&0&1&0
\end{array}\right]
\left[\begin{array}{ccc|cc|cc|c|c}
0&0&0& & & & & & \\
0&0&0& &0& & & & \\
0&0&0& & & & & & \\
\hline
 & & &0&g_1& &0& & \\
 & & &-g_1&0& & & & \\
\hline
 &0& & & &0&g_2& 0& \\
 & & & & &-g_2&0& & \\
\hline
 & & &0& & 0 & &sC_1&0\\
\hline
 & & & & 0& &0 &0&sC_2
\end{array}\right]
\left[\begin{array}{ccc}
1&0&0\\0&1&0\\0&0&1\\ \hline 1&0&0\\0&0&1\\0&0&1\\0&1&0\\0&0&1\\1&-1&0
\end{array}\right]
$$

$$
= \begin{array}{c}
\begin{array}{ccccccccc}1&2&3&4&5&6&7&8&9\end{array}\\
\left[\begin{array}{ccccccccc}
0&0&0&0&g_1&0&0&0&sC_2\\
0&0&0&0&0&-g_2&0&0&-sC_2\\
0&0&0&-g_1&0&0&g_2&sC_1&0
\end{array}\right]
\end{array}
\times
\left[\begin{array}{ccc}
1&0&0\\0&1&0\\0&0&1\\ \hline 1&0&0\\0&0&1\\0&0&1\\0&1&0\\0&0&1\\1&-1&0
\end{array}\right]
$$

$$
= \left[\begin{array}{cc|c}
sC_2 & -sC_2 & g_1\\
-sC_2 & sC_2 & -g_2\\
\hline
-g_1 & g_2 & sC_1
\end{array}\right] = Y_{txf}
\qquad
\begin{bmatrix}-j_1\\-j_2\\-j_3\end{bmatrix} = -\begin{bmatrix} i_{ports} \end{bmatrix}
$$

For the 2-port set $j_3 = 0$; eliminates inner node

$$
\begin{bmatrix} i_{1\,port}\\ i_{2\,port}\\ 0\end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12}\\ \hline Y_{21} & Y_{22}\end{array}\right]\begin{bmatrix} v_{1t}\\ v_{2t}\\ v_{3t}\end{bmatrix} \Rightarrow \begin{bmatrix} i_1\\ i_2\end{bmatrix}_{port} = Y_{11}\begin{bmatrix} v_1\\ v_2\end{bmatrix}_t + Y_{12}\, v_{3t}
$$

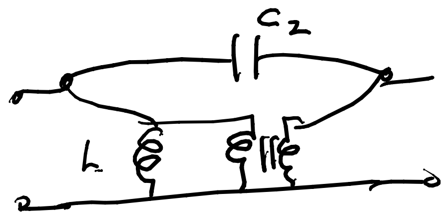
$$
\Rightarrow \quad v_{3t} = Y_{22}^{-1}(-Y_{21})\begin{bmatrix} v_1\\ v_2\end{bmatrix}_t \Rightarrow \begin{bmatrix} i_1\\ i_2\end{bmatrix}_{port} = \left[Y_{11} - Y_{12}Y_{22}^{-1}Y_{21}\right]\begin{bmatrix} v_1\\ v_2\end{bmatrix}_{port}
$$

$$
Y_{2port} = Y_{11} - Y_{12}\cdot Y_{22}^{-1}\, Y_{21}
$$

$$
= \begin{bmatrix} sC_2 & -sC_2\\ -sC_2 & sC_2\end{bmatrix} - \begin{bmatrix} g_1\\ -g_2\end{bmatrix}\frac{1}{sC_1}\begin{bmatrix} -g_1 & g_2\end{bmatrix}
$$

$$= \begin{bmatrix} sC_2 & -sC_2 \\ -sC_2 & sC_2 \end{bmatrix} - \begin{bmatrix} -g_1^2/sC_1 & +g_1 g_2/sC_1 \\ g_1 g_2/sC_1 & -g_2^2/sC_1 \end{bmatrix}$$

$$= sC_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{1}{sC_1} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{bmatrix} = Y_{2\,port}$$

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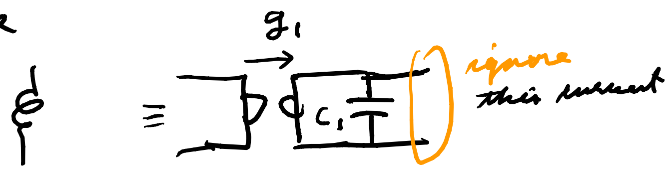


$L = C_1/g_1^2$