EE 303 Midterm Solutions
Fall 2008

#1. $v_o = R i_{D_{Q_1}} - R i_{D_{Q_2}} = R(i_{D_{Q_1}} - i_{D_{Q_2}}) = R(-\alpha I_T \tanh(v_d/(2V_T)))$

(from class notes or p.707 from α((7.72)-(7.73)))

∴ $v_o = -\alpha R I_T \cdot \tanh\left(\frac{v_d}{2V_T}\right)$

#2. M1 is in saturation; $V_{GS} = V_{DS} = V_1 = 3V$

∴ $I_{D_1} = \frac{KP_n}{2}\frac{W}{L}(V_{GS} - VTO_n)^2(1+\lambda V_{DS})$

$= \frac{20.54\times10^{-6}}{2}\cdot\frac{144}{8}(3-1.3)(1+15\times10^{-3}\times 3) = 184.986\times10^{-6}(1.7)(1.045)$

$= 0.329\,mA$

as $v_R = V_{CC} - V_1 = 6-3 = 3 = R\cdot I_{D_1} \Rightarrow R = 3/0.329\times10^{-3}$

a) $\Rightarrow R = 9.12\,k\Omega$

b) as $M_2 = M_1$ when $V_{out} = V_1$, $I_{out} = I_{D_1} = 0.329\,mA$

When $V_{out} = 4 = V_{DS_{M_2}} \Rightarrow I_{out} = I_{D_1}\times\left[\frac{1+\lambda V_{out}}{1+\lambda\cdot V_1}\right] = I_{D_1}\times\left(\frac{1+0.060}{1+0.045}\right)$

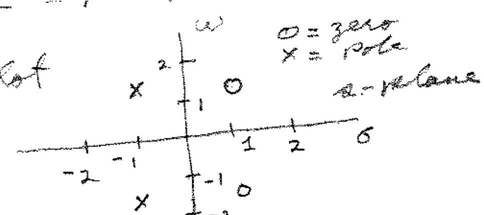
$= I_{D_1}\cdot(1.014)$

∴ $I_{out}\big|_{V_{out}=3} = 0.329\,mA$

$I_{out}\big|_{V_{out}=4} = 0.334\,mA$

#3. $s^2+2s+3=0 \Rightarrow p_{1,2} = -1\pm\frac{1}{2}\sqrt{4-4\times3} = -1\pm j\sqrt{2}$ = poles

$s^2-2s+3=0 \Rightarrow z_{1,2} = +1\pm j\sqrt{2}$ = zeros plot



a) $v_o(t) = T(j2)\,v_i(t) = \frac{5(-2-2\times j2+3)}{(-2+2\times j2+3)}e^{j2t}$

$= 5\left(\frac{1-j4}{1+j4}\right)e^{j2t} = 5e^{j(2t-2\arctan(4/1))}$

$2\times76° = 2.66\,rad$

b) $= 5e^{j(2t-2.66)}$ as $\left|\frac{1-j4}{1+j4}\right| = 1$ (this is all-pass)

as $e^{j2t} = \cos(2t) + j\sin(2t) \Rightarrow \cos(2t) = \frac{e^{j2t}+e^{-j2t}}{2}$

By linearity & real coefficients of T(s)

$v_o(t) = \frac{5}{2}e^{j(2t-2.66)} + \frac{5^*}{2}e^{-j(2t-2.66)}$

c) $= 5\cos(2t-2.66)$

4. $v_o = R_s\, i_{R_S}$ where by KCL: $i_{R_S} = g_m v_{gs} + i_{R_g} + i_{C_g}$

$$= g_m v_{gs} + G_g v_{gs} + s C_g v_{gs}$$

$$= (g_m + G_g + s C_g)\, v_{gs} \quad \text{where } G_g = \frac{1}{R_g}$$

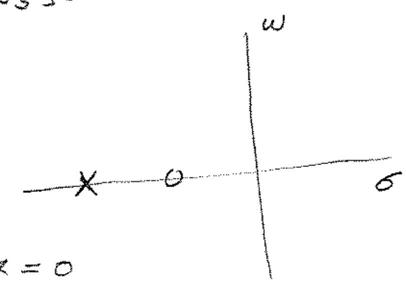
& by KVL: $v_{gs} = v_i - v_o$

$$\therefore \quad v_o = R_s\,(g_m + G_g + s C_g)[v_i - v_o]$$

$$\Rightarrow \quad (1 + R_s[g_m + G_g + s C_g])\, v_o = R_S(g_m + G_g + s C_g)\, v_i$$

$$\Rightarrow \quad \frac{v_o}{v_i} = \frac{R_S(C_g s + [g_m + G_g])}{R_S(C_g s + [g_m + G_g + G_S])} \quad \text{where } G_S = \frac{1}{R_S}$$

Pole: $p = -[g_m + G_g + G_S]/C_g$
zero: $z = -[g_m + G_g]/C_g$
here $|z| < |p| \Rightarrow$



The low frequency gain is for $s = 0$

$$\frac{v_o}{v_i}(0) = \frac{g_m + G_g}{g_m + G_g + G_S} = \frac{g_m + G_g}{g_m + G_g + 1/R_S} = \frac{R_S(g_m + G_g)}{R_S(g_m + G_g) + 1} < 1$$

$$= g_m R_d * \left(\frac{1 + G_g/g_m}{1 + R_S(g_m + G_g)}\right) \quad \text{if } R_S = R_d$$

Here the source follower gain is less than 1 while the grounded source amplifier gain $g_m R_d$ can be large. In all cases the two gains differ by the factor $\left(\frac{1 + G_g/g_m}{1 + R_S(g_m + G_g)}\right)$