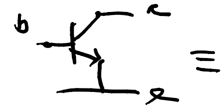
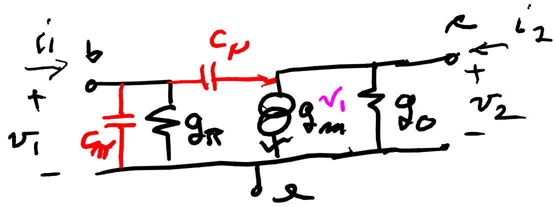
EE 303 Th 09/25/08

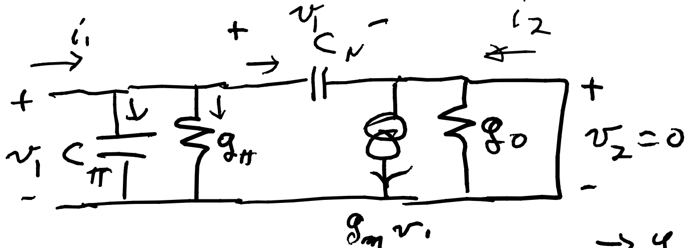




To get the Y matrix: appendix b

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

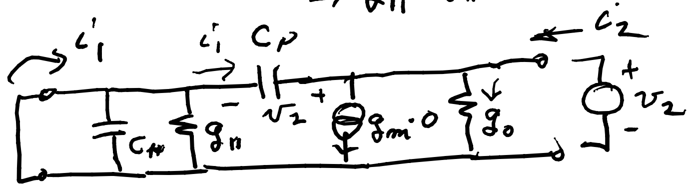
$$i_1 = y_{11} v_1 + y_{12} v_2 \Rightarrow y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0}$$



$$i_1 = i_{C_\pi} + i_{g_\pi} + i_{C_\mu}$$
$$= sC_\pi v_1 + g_\pi v_1 + sC_\mu v_1$$

$$\Rightarrow y_{11} = g_\pi + s(C_\pi + C_\mu)$$

$$y_{12} = \frac{i_1}{v_2}\Big|_{v_1 = 0}$$



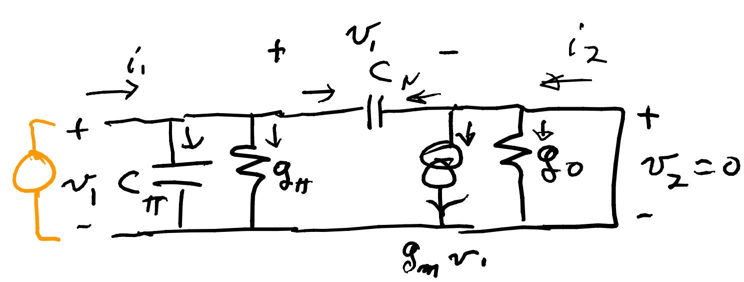
$i_1 = -sC_\mu \cdot v_2$ as $i_{C_\pi} = i_{g_\pi} = 0$ since shorted

$$y_{12} = -sC_\mu$$

$$i_2 = g_0 \cdot v_2 + sC_\mu v_2$$

$$y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0} = g_0 + sC_\mu$$

$$y_{21} = \frac{i_2}{v_1}\Big|_{v_2=0}$$

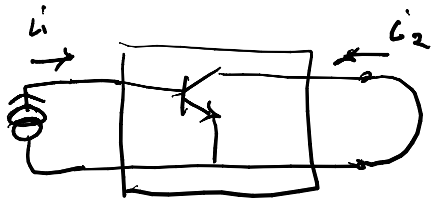


$$i_2\Big|_{v_2=0} = -sC_\mu v_1 + g_m v_1$$

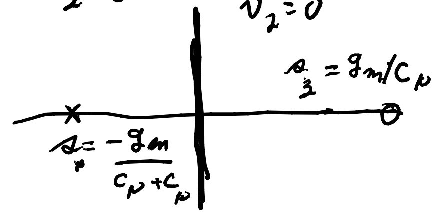
$$i_2 = y_{21} v_1 + \cancel{y_{22} v_2}^{\,0}$$

$$y_{21} = g_m - sC_\mu$$

$$Y = \begin{bmatrix} g_\pi + s(C_\pi + C_\mu) & -sC_\mu \\ g_m - sC_\mu & g_0 + sC_\mu \end{bmatrix}$$



$$\left.\frac{i_2}{i_1}\right|_{v_2=0} = \left.\frac{i_2/v_1}{i_1/v_1}\right|_{v_2=0} = \frac{y_{21}}{y_{11}}$$



$$\frac{i_2}{i_1} = \frac{y_{21}}{y_{11}} = \frac{g_m - sC_\mu}{g_\pi + s(C_\pi + C_\mu)} \simeq \beta(s)$$

$$s = j\omega; \quad \beta(j\omega) = \frac{g_m - jC_\mu\omega}{g_\pi + j\omega(C_\mu + C_\pi)}$$

$$|\beta(j\omega)| = \frac{\sqrt{g_m^2 + (C_\mu\omega)^2}}{\sqrt{g_\pi^2 + \omega^2(C_\mu + C_\pi)^2}}$$

for normal use $C_\mu \omega << g_m$

$$g_m = \frac{|I_c|}{V_T}$$

$$g_\pi = \frac{|I_c|}{\beta_0 V_T} = \frac{g_m}{\beta_0}$$

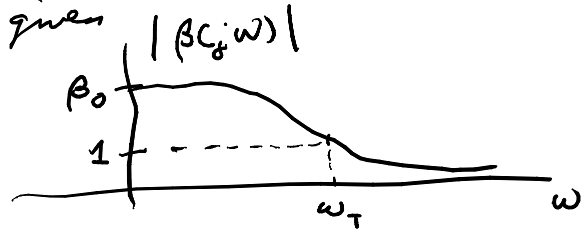
$$\simeq \frac{\sqrt{(g_m/g_\pi)^2}}{\sqrt{1 + \omega^2(C_\mu + C_\pi)^2/g_\pi^2}} = \frac{\beta_0}{\sqrt{1 + \omega^2(C_\mu + C_\pi)^2/g_\pi^2}}$$

look for the frequency ω_T for which $\left|\frac{i_2}{i_1}(j\omega_T)\right| = 1$

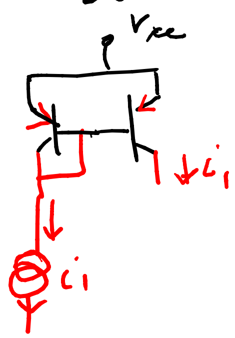
means the transistor becomes ineffective:

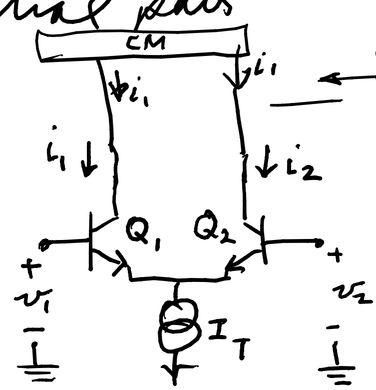
$$1 \simeq \frac{\beta_0 \cdot g_\pi}{\omega(C_\mu + C_\pi)} \Rightarrow \omega_T = \frac{\beta_0 \cdot g_\pi}{(C_\mu + C_\pi)} = \frac{g_m}{C_\mu + C_\pi}$$

In data sheets $f_T = \frac{\omega_T}{2\pi}$ is given



BJT differential pair





$i_o = i_2 - i_1$

bias in the forward active region

$-i_{E_1} - i_{E_2} = I_T$, $i_1 = -\alpha i_{E_1}$, $i_2 = -\alpha i_{E_2}$

$$\frac{I_T}{\alpha^{-1}} = i_1 + i_2 \quad ; \quad i_1 = \alpha I_{ES} e^{v_{BE_1}/V_T}, \quad i_2 = \alpha I_{ES} e^{v_{BE_2}/V_T}$$

$$i_2 - i_1 = i_{out} = \alpha I_{ES}\left(e^{v_{BE_2}/V_T} - e^{v_{BE_1}/V_T}\right)$$

$$= \alpha I_{ES}\, e^{v_{BE_2}/V_T}\left(1 - e^{(v_{BE_1} - v_{BE_2})/V_T}\right)$$

$$\alpha I_T = \alpha I_{ES}\left(e^{v_{BE_2}/V_T} + e^{v_{BE_1}/V_T}\right) = \alpha I_{ES}\, e^{v_{BE_2}/V_T}\left(1 + e^{\frac{(v_{BE_1} - v_{BE_2})}{V_T}}\right)$$

$$v_d = v_{BE_1} - v_{BE_2} \Rightarrow \frac{i_{out}}{I_T} = \frac{1 - e^{v_d/V_T}}{1 + e^{v_d/V_T}} = \frac{e^{v_d/2V_T}}{e^{v_d/2V_T}}\left(\frac{e^{-\frac{v_d}{2V_T}} - e^{v_d/2V_T}}{e^{-v_d/2V_T} + e^{v_d/2V_T}}\right)$$

$$i_{out} = -I \cdot \tanh(v_d/2V_T)$$

$$i_o = -I_T \tanh(v_d/2V_T)$$