ENEE 610
HW IV

Solutions for problem 1 & 2
Problem 3 solution is incomplete.

Problem 1
Part a) and b) are trivial. Let me know if you need the graph.

(c) Reducing Semi-State equations to Steady-State form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ C & 0 \\ 0 & -L \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & +1 \\ 0 & 0 & SC_3 & 0 & 0 & +1 \\ -1 & -1 & 0 & -SL_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -T \\ -T & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \underline{X}$$

$$+ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 S \\ i_2 S \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} C & 0 \\ 0 & -L \end{bmatrix} \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{u}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & T \\ -T & -1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

interchange columns 5&6 with 3&4

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & T & 0 & 0 \\ -T & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{u}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ T & 0 \\ 0 & -T \\ 0 & 1 \end{bmatrix} \underline{u}$$

$$\begin{bmatrix} C_3 & 0 \\ 0 & -L_4 \end{bmatrix} \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \left[\begin{array}{cc|cc|cc} 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{u}$$

$$\Rightarrow \begin{bmatrix} C_3 & 0 \\ 0 & -L_4 \end{bmatrix} \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_6 \end{bmatrix} + \cancel{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}} + \cancel{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u}$$

$$= \begin{bmatrix} 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -T \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ T & 0 \\ 0 & -T \\ 0 & 1 \end{bmatrix} \underline{u}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ T-1 & 0 \end{bmatrix} \underline{u}$$

Assume $C_3 \,\&\, L_4 \neq 0 \Rightarrow$

$$\begin{bmatrix} C_3 & 0 \\ 0 & -L_4 \end{bmatrix}^{-1} = \frac{1}{-L_4 C_3} \begin{bmatrix} -L_4 & 0 \\ 0 & C_3 \end{bmatrix} = \begin{bmatrix} 1/C_3 & 0 \\ 0 & -1/L_4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 1/C_3 & 0 \\ 0 & -1/L_4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1/C_3 & 0 \\ 0 & -1/L_4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ T-1 & 0 \end{bmatrix} \underline{u}$$

$$\Rightarrow \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1/C_3 \\ -1/L_4 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & -1/C_3 \\ -1/L_4 (T-1) & 0 \end{bmatrix} \underline{u}$$

$$\Rightarrow \boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}} \leftarrow \text{State - equations}$$

$y = v_2 \Rightarrow v_2 = v_1 - v_4$

$v_4 = i_4 s L_4 \quad ; \quad i_4 = x_4$

$$\Rightarrow y = \begin{bmatrix} 0 & -sL_4 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 s \\ i_2 s \end{bmatrix}$$

d) from Lecture Notes we have

$$T(s) = C(sI - A)^{-1}B + D$$

where

$$D = [1 \quad 0]$$

$$C = [0 \quad -sL_4]$$

$$A = \begin{bmatrix} 0 & 1/C_3 \\ -1/L_4 & 0 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0 & -1/C_3 \\ \frac{(T+1)}{L_4} & 0 \end{bmatrix}$$

$$\Rightarrow T = \Bigg[\overbrace{\frac{s^2 C_3 L_4 + 1(1-T)}{(s^2 C_3 \cdot L_4 + 1)} + 1}^{T_1} \quad , \quad \overbrace{\frac{-sL_4}{s^2 C_3 L_4 + 1}}^{T_2} \Bigg]$$

e) for 1^{st} entry poles are $\pm 1 / \sqrt{-L_4 C_3} = \pm \frac{1}{j\sqrt{C_3 L_4}}$

We have the same poles for the second entry of $T(s)$ as well.

Since the poles are not in the RHP $\Rightarrow$ 1^{st} Condition is satisfied

Also the poles are simple, which are the order of $\underline{s}$ or lower.

for the 3^{rd} Condition we need $Re\{\text{each entry}\} \geq 0 \quad \forall\, 0 \leq \omega \leq \infty$

for 1^{st} entry.

$s = \sigma + j\omega$

$$Real\{T_1\} = 1/2\{T_1(s) + T_1^*(s)\} = 1/2\left\{ \frac{(\sigma+j\omega)^2 C_3 L_4 + (1-T)}{s^2 C_3(L_4+1)} ; + 2 + \frac{(\sigma-j\omega)^2 C_3 L_4 + (1-T)}{s^{*2} C_3(L_4+1)\, C_3(1-T} \right.$$

• No poles in RHP ⇒ $s \to j\omega$)

$$= \tfrac{1}{2}\left\{ \frac{-\omega^2(C_3L_4+(1-T))(-\omega^2)}{s^2C_3(L_4+1)} - \frac{+-\omega^2C_3L_4(1-T)(-\omega)^2 + 2(\omega^2)C_3(L_4+1)}{s^{*2}C_3(L_4+1)} \right\}$$

$$= \tfrac{1}{2}\left\{ \frac{2\omega^2C_3L_4+2\omega^4(1-T) + 2\omega^2C_3(L_4+1)}{s^2C_3(L_4+1)} \right\}$$

⇒ The numerator of the above is positive for $0 \leq \omega < \infty$

⇒ 1^{st} entery of $T(s)$ is <u>Positive Real</u>

for 2^{nd} entery we have:

$$Re\{T_2(s)\} = \tfrac{1}{2}\{T_2(s)+T_2^*(s)\} = \tfrac{1}{2}\left\{ \frac{-(\sigma+j\omega)L_4}{s^2C_3(L_4+1)} + \frac{-(\sigma-j\omega)L_4}{s^{*2}C_3(L_4+1)} \right\}$$

$$= \tfrac{1}{2}\,\frac{-L_4(\sigma+j\omega)(\sigma-j\omega)^2 + (-\sigma+j\omega)(\sigma+j\omega)^2L_4}{s^2 \cdot s^{*2}\,C_3(L_4+1)L_4}$$

$$= \tfrac{1}{2}\,\frac{-(\sigma+j\omega)(\sigma^2-2\sigma j\omega-\omega^2) + (-\sigma+j\omega)(\sigma^2+2j\sigma\omega-\omega^2)}{s^2 \cdot s^{*2}\,C_3(L_4+1)L_4}$$

$$= \tfrac{1}{2}\,\frac{(-\sigma^3 - \cancel{2\sigma^2 j\omega} + \cancel{\sigma\omega^2} - \cancel{j\omega\sigma^2} - \sigma\omega^2 + \cancel{j\omega^3} - \sigma^3 - \cancel{2j\sigma^2\omega} - \cancel{\sigma\omega^2} + \cancel{j\omega\sigma^2} - 2\sigma\omega^2 - \cancel{j\omega^3})(L_4)}{s^2 \cdot s^{*2}\,C_3(L_4+1)L_4}$$

$$= \tfrac{1}{2}\left(\frac{(-2\sigma^3 - 3\sigma\omega^2)L_4}{(\cdot)}\right)$$ ⇒ This is negative for all $0 \leq \omega < \infty$

⇒ Second Term in $T(s)$ is <u>not real Positive</u>

Problem 2

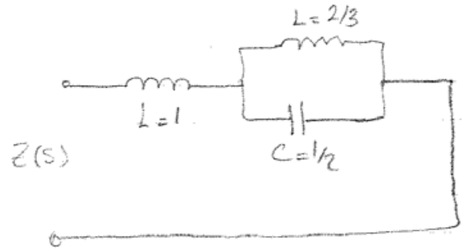
Problem 2)

$Z(S) = S(S^2+5) / (S^2+3)$

1st Foster Method: $Z(S) = \dfrac{S^3+5S}{S^2+3} = S + \dfrac{2S}{S^2+3}$

So we have $K_\infty = 1$, $2K_2 = 2$, $\omega_2^2 = 3$

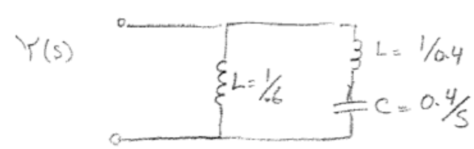
$\Rightarrow$



So for 2nd Foster Method:

$Y(S) = \dfrac{1}{Z(S)} = \dfrac{S^2+3}{S(S^2+5)} = \dfrac{.6}{S} + \dfrac{.4S}{S^2+5}$

$\Rightarrow$



For Cauer Method:

1^{st} Cauer Method:

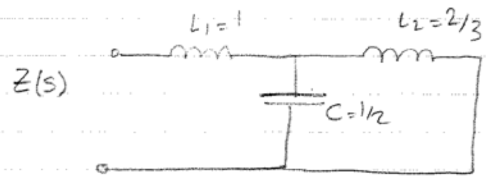
we have $Z(s) = \frac{s(s^2+5)}{s^2+3} = s + \frac{2s}{s^2+3}$.

So $\frac{2s}{s^2+3} = Z_1$ & $\underline{L_1 = 1}$

$\Rightarrow$ So, $Y_1(s) = \frac{1}{Z_1} = \frac{s^2+3}{2s} = \frac{1}{2}s + \frac{3}{2s} \Rightarrow \underline{C = 1/2}$

and $Z_2 = \frac{1}{Y_2} = \frac{2s}{3} \Rightarrow \underline{L_2 = 2/3}$

$\Rightarrow$



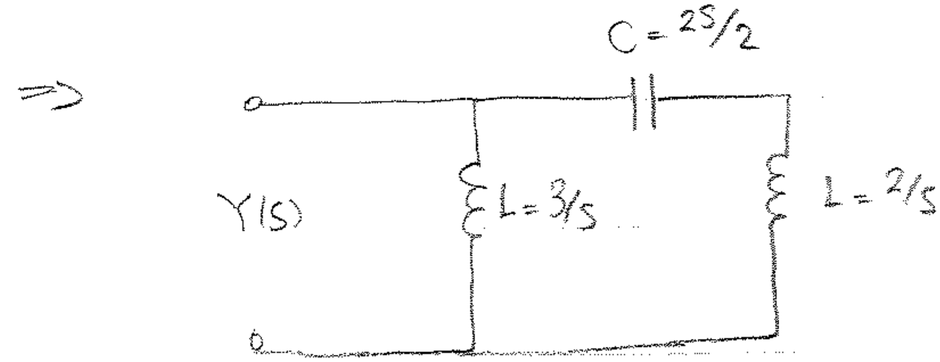
for 2^{nd} Cauer Method:

$Z(s) = \frac{s(s^2+5)}{s^2+3} \Rightarrow Y(s) = \frac{s^2+3}{s^3+5s}$

$\Rightarrow Y(s) = \frac{3}{5s} + \frac{(2/5)s^2}{s^3+5s} \Rightarrow \boxed{\underline{L = 3/5}}$

So $Z = \frac{s^3+5s}{2/5\, s^2} = \frac{25}{2s} + \frac{s}{(2/5)} \Rightarrow C = 25/2$

$\Rightarrow Y = \frac{2/5}{s} \Rightarrow \underline{L_2 = 2/5}$



Problem 3
Suggestion: I will start by writing the real part of the ratio as the ratio of the real-parts PLUS the ratio of the imaginary parts. Then decompose each term into its REAL and IMAG. I am not sure if this leads to the solution!