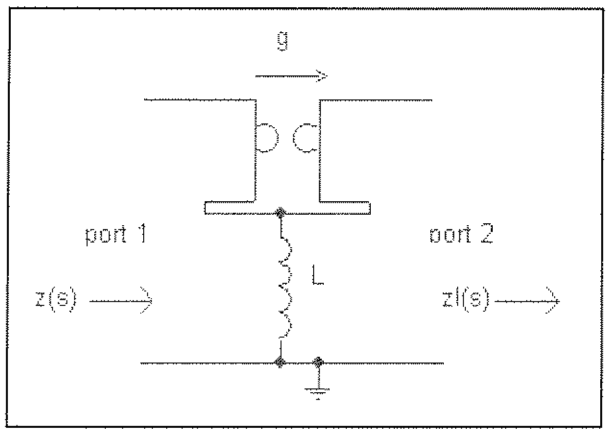
ENEE 610
Homework III
Solutions

Problem 1.



Part a)

In order to find the relation between the Zl and Zin, we have to first derive the 2-port impedance matrix. This can be done simply by writing the indefinite impedance/admittance matrix and grounding one node and elimination of the internal nodes. We have:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{g^2}\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} sL & sL \\ sL & sL \end{bmatrix} = \begin{bmatrix} sL & sL+\frac{1}{g} \\ sL-\frac{1}{g} & sL \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Then we have the 2-port impedance matrix. From the lectures we have:

$$Z_{IN} = \frac{\Delta Z + Z_{11} Z_l}{Z_{22} + Z_l} \qquad Z_l = \frac{\Delta Z - Z_{IN} Z_{22}}{Z_{IN} - Z_{11}}$$

Where Zl is the load impedance, Zin is the input impedance and DeltaZ is the determinant of the 2-port impedance matrix.

Then we have

$$Z_{11} = sL$$

$$Z_{12} = sL + \frac{1}{g}$$

$$Z_{21} = sL - \frac{1}{g},$$

$$Z_{22} = sL$$

$$\boxed{Z_\ell = \frac{\frac{1}{g^2} - Z_{IN}\, sL}{Z_{IN} - sL}}$$

$$\Delta Z = \frac{1}{g^2}$$

Part b)
Richard Function:
Having found the Zl(s) we will do the followings:

We write down the equation for the Richard;s function (General form)
We start modify the generic form of the Richard function (same as lectures)
We will multiply the Zl(s) with **g** to make a dimension less ration
We compare the two rations (one is the modified R(s) and other is **g.Zl(s)**
We will read-off **K,** and **Z(k)**

Here is the work:
Here is the Richard function

$$R(\Delta) = \frac{k Z(\Delta) - \Delta Z(k)}{k Z(k) - \Delta Z(\Delta)}$$

And the modified Richard's function

$$\frac{1}{R(\Delta)} = \frac{k Z(k) - \Delta Z(\Delta)}{k Z(\Delta) - \Delta Z(k)} \Rightarrow \frac{k Z(k)}{k Z(k)} \left(\frac{1 - \frac{\Delta}{k} \frac{Z(\Delta)}{Z(k)}}{\frac{Z(\Delta)}{Z(k)} - \frac{\Delta}{k}} \right)$$

From before we had the Zl(s). Now if we multiply it by **g** we have the following:

$$Z\ell g = \frac{1 - Z_{IN}\, sLg^2}{g Z_{IN} - sLg}$$

Then, if we equate these two, we have:

$$\frac{1 - Z_{IN} s L g^2}{g Z_{IN} - s L g} = \frac{1 - \frac{s}{K}\frac{Z(s)}{Z(k)}}{\frac{Z(s)}{Z(k)} - \frac{s}{K}}$$

And from that, we can find K, and Z(k)

For K we have:

$$K = \frac{1}{gL}$$

And for Z(k):

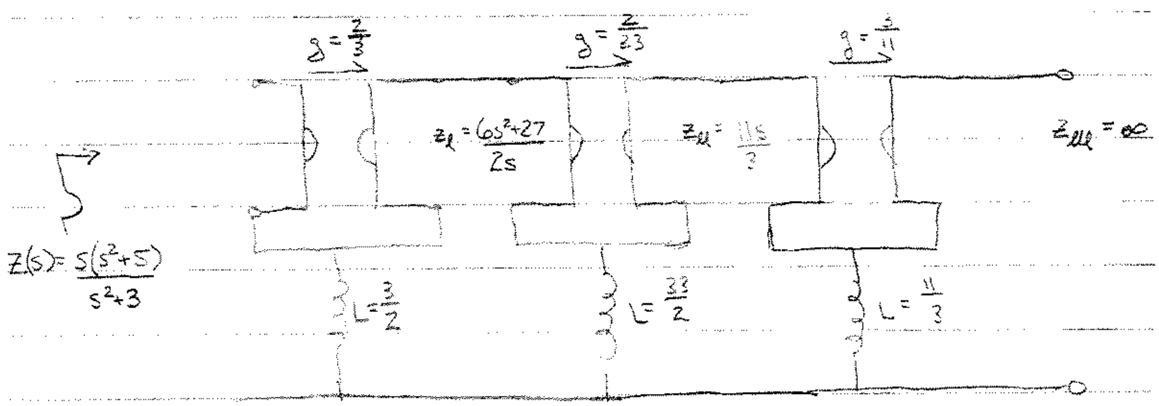
$$L = \frac{Z(k)}{K}$$

Subbing for K, and Z(k) in Zl(s) we have:

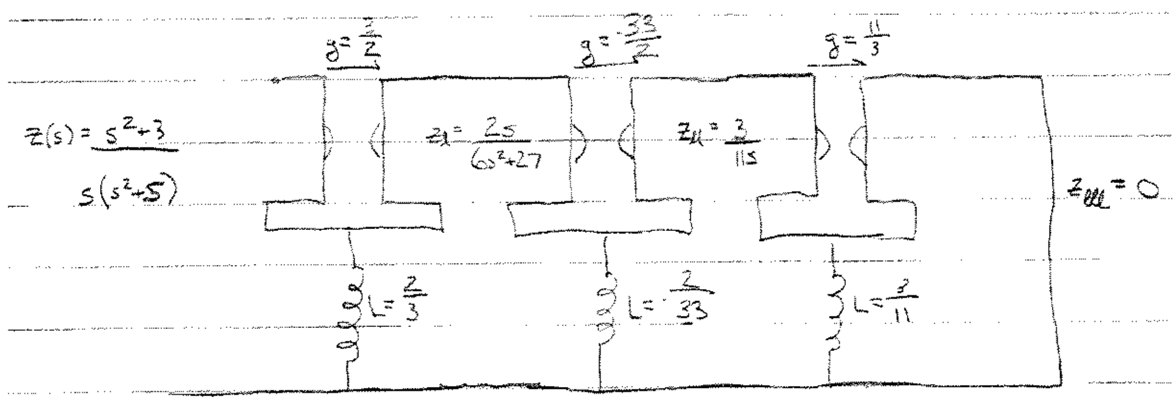
$$\boxed{Z_l(s) = \frac{1}{g Z_r(s)}} \quad \text{WHERE} \quad \boxed{Z_r(s) = \frac{k Z(s) - s Z(k)}{k Z(k) - s Z(s)}}$$

Part c)

For the following section, all we have to do, is to decompose the given functions as a series of Richard's functions above. And then simply cascading the circuits in part a) to synthesis the circuit.



Synthesis for C1



Synthesis for C2

Calculations for C1:

GIVEN $Z(s) = \frac{s(s^2+5)}{s^2+3}$ SHOW $K=1$ IS A ZERO $\Rightarrow$ TRUE IF $K[Z(-K)+Z(K)]=0$

$Z(1) = \frac{1\cdot(1^2+5)}{1^2+3} = \frac{6}{4} = \frac{3}{2}$ & $Z(-1) = \frac{-1\cdot((-1)^2+5)}{(-1)^2+3} = \frac{-6}{4} = \frac{-3}{2} \Rightarrow 1\left[-\frac{3}{2}+\frac{3}{2}\right]=0$

$\therefore$ $K=1$ IS A ZERO OF THE EVEN PART FOR $Z(s) = \frac{s(s^2+5)}{s^2+3} = Z_{IN}$

GIVEN $K=1$; $g = \frac{1}{Z(K)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$ & $L = \frac{Z(K)}{K} = \frac{\frac{3}{2}}{1} = \frac{3}{2}$

$$Z_L(s) = \frac{\frac{1}{g^2} - Z_{IN}\,sL}{Z_{IN} - sL} \Rightarrow \frac{\frac{1}{\left(\frac{2}{3}\right)^2} - \left[\frac{s(s^2+5)}{s^2+3}\right] s\cdot\frac{3}{2}}{\left[\frac{s(s^2+5)}{s^2+3}\right] - s\cdot\frac{3}{2}} = \frac{\frac{9}{4}(s^2+3) - \frac{3}{2}\left(s^2(s^2+5)\right)}{s(s^2+5) - \frac{3}{2}s(s^2+3)}$$

$$= \frac{\frac{9}{4}s^2 + \frac{27}{4} - \frac{3}{2}s^4 - \frac{15}{2}s^2}{s^3+5s - \frac{3}{2}s^3 - \frac{9}{2}s} = \frac{-\frac{3}{2}s^4 - \frac{21}{4}s^2 + \frac{27}{4}}{-\frac{1}{2}s^3 + \frac{1}{2}s} = \frac{-6s^4 - 21s^2 + 27}{-2s^3 + 2s}$$

GIVEN $Z_{r}(s) = \frac{6s^2+27}{2s}$ & $k=1$; $Z(1) = \frac{6(1)^2+27}{2(1)} = \frac{33}{2}$ $Z(-1) = \frac{6(-1)^2+27}{2(-1)} = \frac{-33}{2}$

$g = \frac{1}{Z(k)} = \frac{1}{\frac{33}{2}} = \frac{2}{33}$ & $L = \frac{Z(k)}{k} = \frac{\frac{33}{2}}{1} = \frac{33}{2}$

$$\frac{1}{Z_r(s)} = \frac{kZ(k) - sZ(s)}{kZ(s) - sZ(k)} = \frac{1\left(\frac{33}{2}\right) - s\left[\frac{6s^2+27}{2s}\right]}{1\left[\frac{6s^2+27}{2s}\right] - s\left(\frac{33}{2}\right)} = \frac{33s - s(6s^2+27)}{(6s^2+27) - s(s(33))} = \frac{33s - 6s^3 - 27s}{6s^2+27-33s^2}$$

$$= \frac{-6s^3+6s}{-27s^2+27} = \frac{-\overset{2}{6}s\,\cancel{(s^2-1)}}{+\underset{9}{27}\,\cancel{(s^2-1)}} = \frac{2s}{9} \qquad Z_{ll}(s) = \frac{1}{g}\cdot\frac{1}{Z_r(s)} = \frac{33}{2}\cdot\frac{2s}{9} = \frac{11s}{3}$$

GIVEN $Z_{ll}(s) = \frac{11s}{3}$ & $k=1$; $Z(1) = \frac{11}{3}$ $Z(-1) = \frac{-11}{3}$ $g = \frac{1}{Z(k)} = \frac{3}{11}$ $L = \frac{Z(k)}{k} = \frac{\frac{11}{3}}{1} = \frac{11}{3}$

$$\frac{1}{Z_r(s)} = \frac{kZ(k) - sZ(s)}{kZ(s) - sZ(k)} = \frac{1\left(\frac{11}{3}\right) - s\left(\frac{11s}{3}\right)}{1\left(\frac{11s}{3}\right) - s\left(\frac{11}{3}\right)} = \frac{1-s^2}{s-s} \Rightarrow \infty$$

Similarly for C2 we have:

k2) GIVEN $Z(s) = \dfrac{s^2+3}{s(s^2+5)}$ SHOW $k=1$ IS A ZERO $\Rightarrow$ TRUE IF $k\left[Z(-k)+Z(k)\right] = 0$

$$Z(1) = \frac{1^2+3}{1\cdot((1)^2+5)} = \frac{4}{6} \quad \& \quad Z(-1) = \frac{(-1)^2+3}{-1\cdot(-1)^2+5} = \frac{-4}{6} \Rightarrow 1\left[\frac{-4}{6}+\frac{4}{6}\right] = 0 \checkmark$$

$\therefore$ $k=1$ IS A ZERO OF THE EVEN PART FOR $Z(s) = \dfrac{s^2+3}{s(s^2+5)} = Z_{IN}$

GIVEN $k=1$; $g = \dfrac{1}{Z(k)} = \dfrac{1}{\frac{4}{6}} = \dfrac{3}{2}$; $L = \dfrac{Z(k)}{k} = \dfrac{\frac{2}{3}}{1} = \dfrac{2}{3}$

$$Z_\ell(s) = \frac{\frac{1}{g^2} - Z_{IN}sL}{Z_{IN} - sL} \Rightarrow \frac{\frac{1}{\left(\frac{3}{2}\right)^2} - \left[\frac{s^2+3}{s(s^2+5)}\right] s\cdot\frac{2}{3}}{\left[\frac{s^2+3}{s(s^2+5)}\right] - s\cdot\frac{2}{3}} = \frac{\frac{4}{9}\left(s(s^2+5)\right) - \frac{2}{3}s\left(s^2+3\right)}{(s^2+3) - \frac{2}{3}s\left(s\left(s^2+5\right)\right)}$$

$$= \frac{\frac{4}{9}s^3 + \frac{20}{9}s - \frac{2}{3}s^3 - 2s}{s^2+3-\frac{2}{3}s^4 - \frac{10}{3}s^2} = \frac{(4-6)s^3 + (20-18)s}{-6s^4 + (9-30)s^2 + 27} = \frac{-2s^3+2s}{-6s^4-21s^2+27} = \frac{+2s\cancel{(s^2-1)}}{\cancel{(s^2-1)}(6s^2+27)}$$

$$Z_\ell(s) = \frac{2s}{6s^2+27}$$

1c2 cont) GIVEN $Z_l(s) = \frac{2s}{6s^2+27}$ & $k=1$ $\quad z(1) = \frac{2(1)}{6(1)^2+27} = \frac{2}{33} \quad z(-1) = \frac{2(-1)}{6(-1)^2+27} = \frac{-2}{33}$

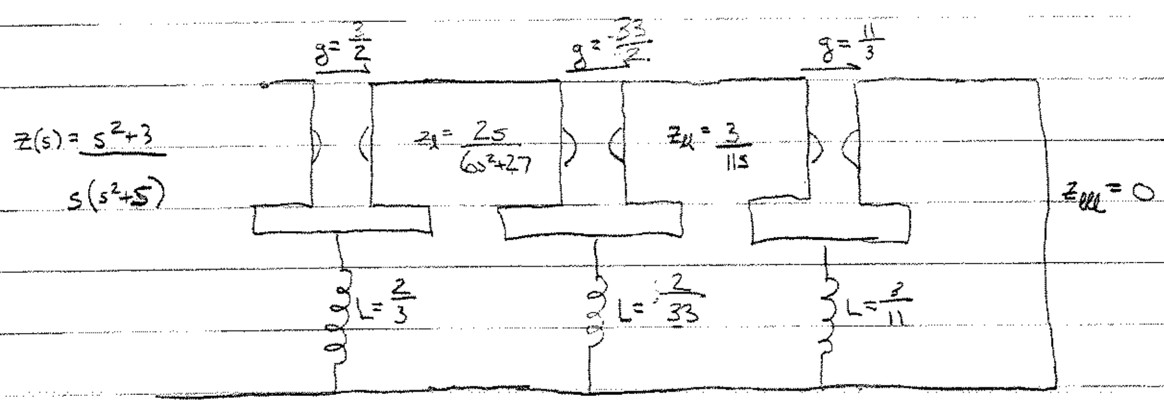
$$g = \frac{1}{z(k)} = \frac{1}{\frac{2}{33}} = \frac{33}{2}, \quad L = \frac{z(k)}{k} = \frac{\frac{2}{33}}{1} = \frac{2}{33}$$

$$\frac{1}{Z_r(k)} = \frac{k z(k) - s z(s)}{k z(s) - s z(k)} = \frac{1\left(\frac{2}{33}\right) - s\left[\frac{2s}{6s^2+27}\right]}{1\left[\frac{2s}{6s^2+27}\right] - s\left(\frac{2}{33}\right)} = \frac{\cancel{2}(6s^2+27) - \cancel{2}s^2(33)}{33\cdot(\cancel{2}s) - \cancel{2}s(6s^2+27)} = \frac{6s^2+27-33s^2}{33s-6s^3-27s}$$

$$= \frac{-27s^2+27}{-6s^3+6s} = \frac{+27(s^2-1)}{+6s\cancel{(s^2-1)}} = \frac{9}{2s} \qquad z_{ll}(s) = \frac{1}{g}\cdot\frac{1}{Z(s)} = \frac{\cancel{2}}{33}\cdot\frac{9}{\cancel{2}s} = \frac{3}{11s}$$

GIVEN $z_{ll}(s) = \frac{3}{11s}$ & $k=1$; $\quad z(1) = \frac{3}{11}$; $\quad z(-1) = \frac{-3}{11} \qquad g = \frac{1}{z(k)} = \frac{11}{3} \qquad L = \frac{z(k)}{k} = \frac{\frac{3}{11}}{1} = \frac{3}{11}$

$$\frac{1}{Z_r(s)} = \frac{k z(k) - s z(s)}{k z(s) - s z(k)} = \frac{1\left(\frac{3}{11}\right) - \cancel{s}\left(\frac{3}{11\cancel{s}}\right)}{1\left(\frac{3}{11s}\right) - s\left(\frac{3}{11}\right)} = \frac{\frac{3}{11} - \frac{3}{11}}{\frac{3}{11s} - \frac{3s}{11}} = 0 \qquad Z_{lll}(s) = 0$$



1d) GIVEN $z(s) = \dfrac{s(s^2+5)}{(s^2+3)}$ SHOW $k=2$ IS A ZERO $z(2) = \dfrac{2((2)^2+5)}{(2)^2+3} = \dfrac{18}{7}$ $z(-2) = \dfrac{-2((-2)^2+5)}{(-2)^2+3} = \dfrac{-18}{7}$

SINCE $2\left[\dfrac{-18}{7} + \dfrac{18}{7}\right] = 0$; $k=2$ IS A ZERO OF $z(s)$

$k=2$; $g = \dfrac{1}{z(k)} = \dfrac{1}{\frac{18}{7}} = \dfrac{7}{18}$; $L = \dfrac{z(k)}{k} = \dfrac{\frac{18}{7}}{2} = \dfrac{9}{7}$

$$\frac{1}{z_r(s)} = \frac{kz(k) - sz(s)}{kz(s) - sz(k)} \Rightarrow \frac{2\cdot\left(\frac{18}{7}\right) - s\cdot\left[\frac{s(s^2+5)}{s^2+3}\right]}{2\left[\frac{s(s^2+5)}{s^2+3}\right] - s\cdot\frac{18}{7}} = \frac{\frac{36}{7}(s^2+3) - s(s(s^2+5))}{2s(s^2+5) - \frac{18}{7}s(s^2+3)}$$

$$= \frac{\frac{36}{7}s^2 + \frac{108}{7} - s^4 - 5s^2}{2s^3 + 10s - \frac{18}{7}s^3 - \frac{54}{7}s}\cdot\left(\frac{-7}{-7}\right) = \frac{7s^4 + (+35-36)s^2 - 108}{(-14+18)s^3 + (54-70)s} = \frac{7s^4 - s^2 - 108}{4s^3 - 16s} = \frac{\cancel{(s^2-4)}(7s^2+27)}{4s\,\cancel{(s^2-4)}}$$

$$z_\ell(s) = \frac{1}{gz_r(s)} = \frac{1}{\frac{7}{18}}\cdot\frac{7s^2+27}{4s} = \frac{9\cancel{18}}{7}\cdot\frac{7s^2+27}{\cancel{2}4s} \Rightarrow \boxed{\frac{63s^2+243}{14s} = z_\ell(s)}$$

GIVEN $z_\ell(s) = \dfrac{63s^2+243}{14s}$ & $k=2$ $z(2) = \dfrac{63(2)^2+243}{14(2)} = \dfrac{495}{28}$ $z(-2) = \dfrac{63(-2)^2+243}{14(-2)} = \dfrac{-495}{28}$

$g = \dfrac{1}{z(k)} = \dfrac{1}{\frac{495}{28}} = \dfrac{28}{495}$ $L = \dfrac{z(k)}{k} = \dfrac{\frac{495}{28}}{2} = \dfrac{495}{56}$

$$\frac{1}{z_r(s)} = \frac{kz(k) - sz(s)}{kz(s) - sz(k)} = \frac{2\left[\frac{495}{28}\right] - s\left[\frac{63s^2+243}{14s}\right]}{2\left[\frac{63s^2+243}{14s}\right] - s\left[\frac{495}{28}\right]}\;28s = \frac{2s\cdot 495 - 2s(63s^2+243)}{4\cdot(63s^2+243) - s^2(495)} = \frac{990s - 126s^3 - 486s}{252s^2 + 972 - 495s^2}$$

1d cont) $\dfrac{1}{Z_r(s)} = \dfrac{-126s^3 + 504s}{-243s^2 + 972} = \dfrac{+126s(s^2-4)}{+243(s^2-4)} = \dfrac{126s}{243}$

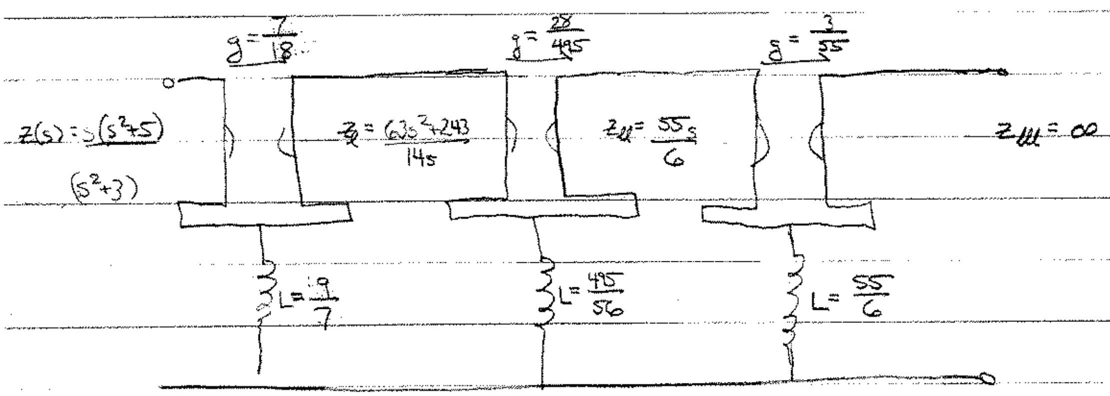
$Z_{ll}(s) = \dfrac{1}{g} \cdot \dfrac{1}{Z_r(s)} = \dfrac{1}{\frac{28}{495}} \cdot \dfrac{126s}{243} = \dfrac{\overset{55}{\cancel{495}}}{{}_2\cancel{28}} \cdot \dfrac{\overset{9}{\cancel{126s}}}{\cancel{243}_{27}} = \dfrac{55s}{6}$

GIVEN $Z_{ll}(s) = \dfrac{55s}{6}$ & $k=2$ $\quad z(2) = \dfrac{55(2)}{6} = \dfrac{55}{3}$ $\quad z(-2) = \dfrac{55(-2)}{6} = \dfrac{-55}{3}$

$g = \dfrac{1}{z(k)} = \dfrac{1}{\frac{55}{3}} = \dfrac{3}{55}$ $\quad L = \dfrac{z(k)}{k} = \dfrac{\frac{55}{3}}{2} = \dfrac{55}{6}$

$\dfrac{-1}{Z_r(s)} = \dfrac{kz(k) - sz(s)}{kz(s) - sz(k)} = \dfrac{2 \cdot \left[\frac{55}{3}\right] - s\left[\frac{55s}{6}\right]}{2 \cdot \left[\frac{55s}{6}\right] - s\left[\frac{55}{3}\right]} = \dfrac{4 \cdot 55 - s(55s)}{2(55s) - 2s(55)} = \dfrac{220 - 55s^2}{\cancel{110s - 110s}\;0} \Rightarrow \infty$

$\therefore Z_{lll}(s) = \infty \Rightarrow$ OPEN CIRCUIT



1d2) GIVEN $Z(s) = \frac{s(s^2+5)}{(s^2+3)}$ SHOW $k=-1$ IS A ZERO; PROVEN FOR $k=1$ IN 1c1 $\therefore$ ALSO TRUE FOR $k=-1$

$$Z(-1) = \frac{-1\cdot((-1)^2+5)}{(-1)^2+3} = \frac{-6}{4} = \frac{-3}{2}$$

GIVEN $k=-1$; $g = \frac{1}{Z(k)} = \frac{1}{\frac{-3}{2}} = \frac{-2}{3}$; $L = \frac{Z(k)}{k} = \frac{\frac{-3}{2}}{-1} = \frac{3}{2}$

$$\frac{1}{Z_r(s)} = \frac{kZ(k) - sZ(s)}{kZ(s) - sZ(k)} \rightarrow \frac{-1\cdot\left(\frac{-3}{2}\right) - s\left[\frac{s(s^2+5)}{(s^2+3)}\right]}{-1\left[\frac{s(s^2+5)}{s^2+3}\right] - s\cdot\left(\frac{-3}{2}\right)} = \frac{\frac{3}{2}(s^2+3) - s(s(s^2+5))}{-s(s^2+5) + \frac{3}{2}s(s^2+3)}$$

$$\frac{\frac{3}{2}s^2 + \frac{9}{2} - s^4 - 5s^2}{-s^3 - 5s + \frac{3}{2}s^3 + \frac{9}{2}s}\cdot\left(\frac{-2}{-2}\right) = \frac{2s^4 + (10-3)s^2 - 9}{(2-3)s^3 + (10-9)s} = \frac{2s^4 + 7s^2 - 9}{-s^3 + s} = \frac{\cancel{(s^2-1)}(2s^2+9)}{-s\cancel{(s^2-1)}}$$

$$Z_\ell(s) = \frac{1}{g\,Z_r(s)} = \frac{1}{\frac{-2}{3}}\cdot\boxed{\frac{2s^2+9}{-s}} \Rightarrow \boxed{\frac{6s^2+27}{2s} = Z_\ell(s)}$$

GIVEN $Z_\ell(s) = \frac{6s^2+27}{2s}$ & $k=-1$ $\quad Z(-1) = \frac{6(-1)^2+27}{2(-1)} = \frac{-33}{2}$ $\quad Z(1) = \frac{6(1)^2+27}{2(1)} = \frac{33}{2}$

$g = \frac{1}{Z(k)} = \frac{1}{\frac{-33}{2}} = \frac{-2}{33}$ $\qquad L = \frac{Z(k)}{k} = \frac{\frac{-33}{2}}{-1} = \frac{33}{2}$

$$\frac{1}{Z_r(k)} = \frac{kZ(k) - sZ(s)}{kZ(s) - sZ(k)} = \frac{(-1)\left[\frac{-33}{2}\right] - s\left[\frac{6s^2+27}{2s}\right]}{(-1)\left[\frac{6s^2+27}{2s}\right] - s\left[\frac{-33}{2}\right]} = \frac{33s - 6s^3 - 27s}{-6s^2 - 27 + 33s^2} = \frac{-6s^3 + 6s}{27s^2 - 27}$$

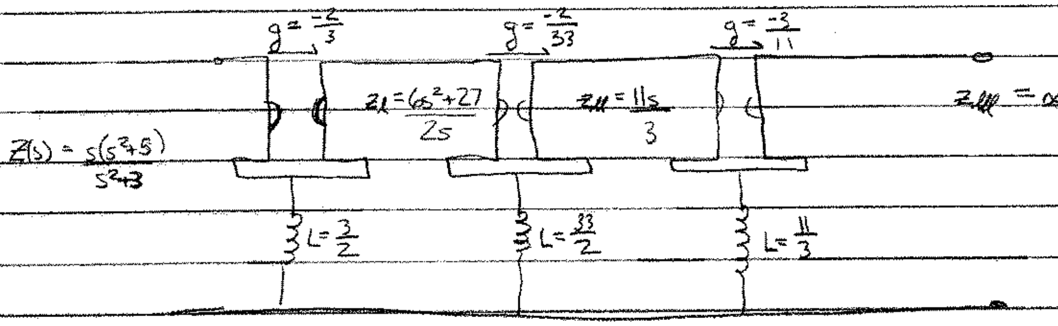
$$= \frac{-6s\cancel{(s^2-1)}}{27\cancel{(s^2-1)}} = \frac{-6s}{27} \qquad Z_{\ell\ell}(s) = \frac{1}{g}\cdot\frac{1}{Z_r(s)} = \frac{+\cancel{33}^{11}}{2}\cdot\frac{+6s}{\cancel{27}_{9}} = \boxed{\frac{11s}{3} = Z_{\ell\ell}(s)}$$

1d2 CONT) GIVEN $Z_{III}(s) = \frac{11s}{3}$ & $k = -1$ $\quad Z(-1) = \frac{-11}{3} \quad Z(1) = \frac{11}{3}$

$g = \frac{1}{Z(k)} = \frac{-3}{11} \qquad L = \frac{Z(k)}{k} = \frac{\frac{-11}{3}}{-1} = \frac{11}{3}$

$$\frac{1}{Z(k)} = \frac{kZ(k) - sZ(s)}{kZ(s) - sZ(k)} = \frac{(-1)\left(\frac{-11}{3}\right) - s\left(\frac{11s}{3}\right)}{(-1)\left(\frac{11s}{3}\right) - s\left(\frac{-11}{3}\right)} = \frac{\frac{11}{3} - \frac{11s^2}{3}}{\underbrace{\frac{-11s}{3} + \frac{11s}{3}}_{\to 0}} \Rightarrow \infty$$

$\therefore Z_{IIII}(s) = \infty \Rightarrow$ OPEN CIRCUIT



1e) A CIRCUIT IS CONSIDERED PASSIVE IF $E(t) = \int_{-\infty}^{t} \begin{bmatrix} i_1 & i_2 \end{bmatrix} Z \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \geq 0$

$$E(t) = \int_{-\infty}^{t} \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} sL & sL + \frac{1}{g} \\ sL - \frac{1}{g} & sL \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \int_{-\infty}^{t} \begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} sLi_1 + sLi_2 + \frac{i_2}{g} & sLi_1 - \frac{i_1}{g} + sLi_2 \end{bmatrix}$$

$$E(t) = \int_{-\infty}^{t} sLi_1^2 + sLi_1i_2 + \cancel{\frac{i_1i_2}{g}} + sLi_1i_2 - \cancel{\frac{i_1i_2}{g}} + sLi_2^2 = \int_{-\infty}^{t} sL(i_1+i_2)^2 \Rightarrow E(t) = sL\int_{-\infty}^{t} (i_1+i_2)^2 \geq$$

THIS RELATIONSHIP WILL HOLD TRUE FOR ALL NON-NEGATIVE VALUES OF L.

$\Rightarrow E(t) \geq 0$ FOR $L \geq 0$; THIS FURTHER IMPLIES THAT g CAN BE POSITIVE OR NEGATIVE.

BASED ON THE DEFINED CIRCUIT AND THE $Z(s)$ FUNCTIONS TESTED, WE CAN CONCLUDE THAT FOR REAL POSITIVE $Z(s)$ CAN BE REPRESENTED AS CASCADES OF THE CIRCUIT GIVEN ; WHERE THE MINIMUM # OF STAGES REQUIRED IS THE HIGHEST ORDER OF s IN $Z(s)$. THUS FOR

$$Z(s) = \frac{s^n + s^{n-1} + s^{n-2} + \ldots}{s^m + s^{m-1} + s^{m-2} + \ldots}$$

WHERE $n > m$; A MINIMUM OF STAGES $= n$

WHERE $m > n$; A MINIMUM OF STAGES $= m$

1e CONT) SINCE EACH STAGE (IN THE CIRCUIT DEFINED) CONTAINTS 2 COMPONENTS; THE MINIMUM # OF COMPONENTS REQUIRED TO REPRESENT $Z(S)$ AS A CASCADE OF THE DEFINED CIRCUIT IS 2^X (X = THE GREATER OF n OR m)

ASPECTS OF K - FOR THE CIRCUIT GIVEN, THE RELATIONSHIP BETWEEN K, g, L WAS DEFINED IN 1b) AS $K = \frac{1}{gL}$. WE CAN QUICKLY SEE, THEREFORE THAT THE PRODUCT OF THE COMPONENTS ON ANY GIVEN STAGE $= \frac{1}{K}$; FURTHERMORE, SINCE WE HAVE SHOWN THAT IT IS PERMISSABLE TO ACHIEVE POSITIVE OR NEGATIVE g VALUES AND MAINTAIN PASSIVITY; IT IS POSSIBLE TO REALIZE CIRCUITS WHEN USING THE CONJUGATE VALUES OF K WHERE K IS A ZERO OF $EV\{Z(s)\}$. THE EXAMPLE FROM THIS SET IS $K=1$, $K=-1$ → COMPARE CIRCUIT REPRESENTATIONS FROM 1c1) TO 1d2) ⇒ SAME CIRCUIT w/ A CHANGED SIGN ON g.

RELATION BETWEEN $Z(s)$ & $Y(s)$ - BY DEFINITION $Z(s) = \frac{1}{Y(s)}$. FOR THIS PROBLEM, THE ILLUSTRATIVE EXAMPLE IS 1c1 $Z(s) = \frac{1}{Z(s)}$ 1c2. COMPARING THE CIRCUIT REPRESENTATIONS, WE QUICKLY NOTE THE INVERSION OF THE VALUES FOR STAGE COMPONENTS

ALSO, IT IS INTERESTING TO NOTE THAT REGARDLESS OF THE K CHOSEN, THE CIRCUIT REPRESENTATION REMAINS THE SAME. ⇒ COMPONENT VALUES CHANGE, BUT THE STRUCTURE REMAINS. EX. 3 STAGE CASCADE w/ OPEN CIRCUIT FOR $K=1$ & $K=2$ & $K=-1$

LASTLY, WHEN $N(s) > D(s)$ THE CASCADE ENDS w/ OPEN CIRCUIT. WHEN $D(s) > N(s)$ THE CASCADE ENDS w/ A SHORT CIRCUIT

2a) FIND ZEROS OF THE EVEN PART; TRUE IF $f(-s)+f(s)=0$

a1) $f(s)=\frac{s+3}{s+5}$; $f(-s)=\frac{-s+3}{-s+5} \Rightarrow \frac{-s+3}{-s+5}+\frac{s+3}{s+5}=0 \Rightarrow \frac{(-s+3)(s+5)+(-s+5)(s+3)}{(-s+5)(s+5)}$

$\frac{-s^2-\cancel{5s}+\cancel{3s}+15-s^2-\cancel{3s}+\cancel{5s}+15}{-s^2-\cancel{5s}+\cancel{5s}+25}=\frac{-2s^2+30}{-s^2+25}=0 \Rightarrow -2s^2+30=0 \Rightarrow \boxed{s=\pm\sqrt{15}}$

a2) $f(s)=\frac{(s+5)}{(s+3)}$; $f(-s)=\frac{(-s+5)}{-s+3} \Rightarrow \frac{-s+5}{-s+3}+\frac{s+5}{s+3}=0 \Rightarrow \frac{(-s+5)(s+3)+(s+5)(-s+3)}{(-s+3)(s+3)}$

$\frac{-s^2-\cancel{3s}+\cancel{5s}+15-s^2+\cancel{3s}-\cancel{5s}+15}{-s^2-\cancel{3s}+\cancel{3s}+9}=\frac{-2s^2+30}{-s^2+9}=0 \Rightarrow -2s^2+30=0 \Rightarrow \boxed{s=\pm\sqrt{15}}$

a3) $f(s)=\frac{s(s+3)}{(s+5)}$; $f(-s)=\frac{-s(-s+3)}{-s+5} \Rightarrow \frac{-s(-s+3)}{-s+5}+\frac{s(s+3)}{s+5}=0 \Rightarrow \frac{-s(-s+3)(s+5)+s(s+3)(-s+5)}{(-s+5)(s+5)}$

$\frac{(s^2-3s)(s+5)+(s^2+3s)(-s+5)}{-s^2-\cancel{5s}+\cancel{5s}+25}=\frac{\cancel{s^3}+5s^2-3s^2-\cancel{15s}-\cancel{s^3}+5s^2-3s^2+\cancel{15s}}{-s^2+25}=\frac{4s^2}{-s^2+25}=0$

$\Rightarrow 4s^2=0 \Rightarrow \boxed{s=0}$

a4) $f(s)=\frac{(s^2+s+1)}{(s^2+2s+1)}$; $f(-s)=\frac{(-s)^2+(-s)+1}{(-s)^2+2(-s)+1} \Rightarrow \frac{s^2-s+1}{s^2-2s+1}+\frac{s^2+s+1}{s^2+2s+1}=\frac{(s^2-s+1)(s^2+2s+1)+(s^2+s+1)(s^2-2s+1)}{(s^2-2s+1)(s^2+2s+1)}$

$\Rightarrow s^4+\cancel{2s^3}+s^2\cancel{-s^3}-2s^2\cancel{-s}+s^2+\cancel{2s}+1+s^4-\cancel{2s^3}+s^2+\cancel{s^3}-2s^2+\cancel{s}+s^2-\cancel{2s}+1$

$s^4+\cancel{2s^3}+s^2-\cancel{2s^3}-4s^2-\cancel{2s}+s^2+\cancel{2s}+1$

a4 CONT) $\dfrac{2s^4 + \cancel{2s^2} - \cancel{4s^2} + \cancel{2s^2} + 2}{s^4 + 2s^2 - 4s^2 + 1} = \dfrac{2s^4 + 2}{s^4 - 2s^2 + 1} \Rightarrow 2s^4 + 2 = 0 \Rightarrow \boxed{s^4 = -1 = e^{j\pi}}$

$s_1 = e^{j\frac{\pi}{4}}$; $s_2 = e^{j\left(\frac{\pi}{4}+\frac{\pi}{2}\right)} = e^{j\frac{3\pi}{4}}$; $s_3 = e^{j\left(\frac{\pi}{4}+\pi\right)} = e^{j\frac{5\pi}{4}}$; $s_4 = e^{j\left(\frac{\pi}{4}+\frac{3\pi}{2}\right)} = e^{j\frac{7\pi}{4}}$

a5) $f(s) = \dfrac{s^2 - 2s + 1}{s^2 + 2s + 1}$; $f(-s) = \dfrac{(-s)^2 - 2(-s) + 1}{(-s)^2 + 2(-s) + 1} \Rightarrow \dfrac{s^2 + 2s + 1}{s^2 - 2s + 1} + \dfrac{s^2 - 2s + 1}{s^2 + 2s + 1}$

$= \dfrac{(s^2+2s+1)(s^2+2s+1) + (s^2-2s+1)(s^2-2s+1)}{(s^2-2s+1)(s^2+2s+1)} = \dfrac{s^4 + \cancel{2s^3} + s^2 + 2s^3 + 4s^2 + 2s + s^2 + \cancel{2s} + 1 + s^4 - \cancel{2s^3} + s^2 - 2s^3 + 4s^2 - \cancel{2s} + s^2 - \cancel{2s} \ldots}{s^4 + \cancel{2s^3} + s^2 - \cancel{2s^3} - 4s^2 - \cancel{2s} + s^2 + \cancel{2s} + 1}$

$= \dfrac{2s^4 + 12s^2 + 2}{s^4 - 2s^2 + 1} = 0 \Rightarrow 2s^4 + 12s^2 + 2 = 0 \Rightarrow s^4 + 6s^2 + 1 = 0$ USING MATLAB, $\boxed{\begin{array}{l} s = \pm\left[\sqrt{2}-1\right] \\ s = \pm\left[\sqrt{2}+1\right] \end{array}}$

2b) $y(s) = \dfrac{s+3}{s+5}$ FROM a1; CHOOSE $K = \sqrt{15}$ AS A ZERO OF THE EVEN PART

SINCE WE'VE SOLVED FOR $Z_\ell(s)$ IN PART 1, CONVERT $y(s)$ TO $z(s) = \dfrac{s+5}{s+3} = \dfrac{1}{y(s)}$ $\quad z(\sqrt{15}) = \dfrac{\sqrt{15}+\ldots}{\sqrt{15}+\ldots}$

$\dfrac{1}{Z_r(s)} = \dfrac{Kz(k) - sz(s)}{Kz(s) - sz(k)}$ WHERE $K = \sqrt{15}$; $g = \dfrac{1}{z(k)} = \dfrac{\sqrt{15}+3}{\sqrt{15}+5}$; $L = \dfrac{z(k)}{k} = \dfrac{1}{\sqrt{15}} \cdot \dfrac{\sqrt{15}+5}{\sqrt{15}+3} = \dfrac{\sqrt{15}+5}{15+3\sqrt{15}}$

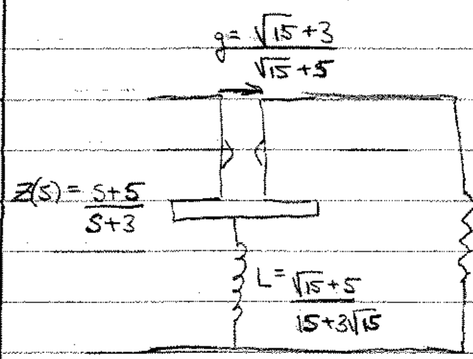
$$\frac{1}{z_r(s)} = \frac{\sqrt{15}\cdot\left(\dfrac{\sqrt{15}+5}{\sqrt{15}+3}\right) - s\cdot\left(\dfrac{s+5}{s+3}\right)}{\sqrt{15}\left(\dfrac{s+5}{s+3}\right) - s\left(\dfrac{\sqrt{15}+5}{\sqrt{15}+3}\right)} = \frac{\sqrt{15}\cdot(\sqrt{15}+5)(s+3) - s(s+5)(\sqrt{15}+3)}{\sqrt{15}(s+5)(\sqrt{15}+3) - s(\sqrt{15}+5)(s+3)}$$

2b CONT) $$\frac{(15+5\sqrt{15})(s+3)+(s^2+5s)(\sqrt{15}+3)}{(s\sqrt{15}+5\sqrt{15})(\sqrt{15}+3)+(-\sqrt{15}s-5s)(s+3)}$$

$$\frac{\cancel{15s}+45+\cancel{5\sqrt{15}s}+15\sqrt{15}-\sqrt{15}s^2-3s^2-\cancel{5\sqrt{15}s}-\cancel{15s}}{\cancel{15s}+\cancel{3\sqrt{15}s}+75+15\sqrt{15}-\sqrt{15}s^2-\cancel{3\sqrt{15}s}-5s^2-\cancel{15s}} = \frac{-(\sqrt{15}+3)s^2+15(\sqrt{15}+3)}{-(\sqrt{15}+5)s^2+15(\sqrt{15}+5)}$$

$$\frac{+(\sqrt{15}+3)(\cancel{s^2+15})}{+(\sqrt{15}+5)(\cancel{s^2+15})} = \frac{1}{Z_r(s)} \Rightarrow Z_\ell(s) = \frac{1}{g}\cdot\frac{1}{Z_r(s)}$$

$$Z_\ell(s) = \frac{\cancel{\sqrt{15}+5}}{\cancel{\sqrt{15}+3}}\cdot\frac{\cancel{\sqrt{15}+3}}{\cancel{\sqrt{15}+5}} = 1$$ SINCE $Y_\ell(s) = \frac{1}{Z_\ell(s)}$ $\boxed{Y_\ell(s)=1}$



$\Rightarrow$