Home Work #2 Solutions

ENEE 610, Fall 07
Prof. Newcomb

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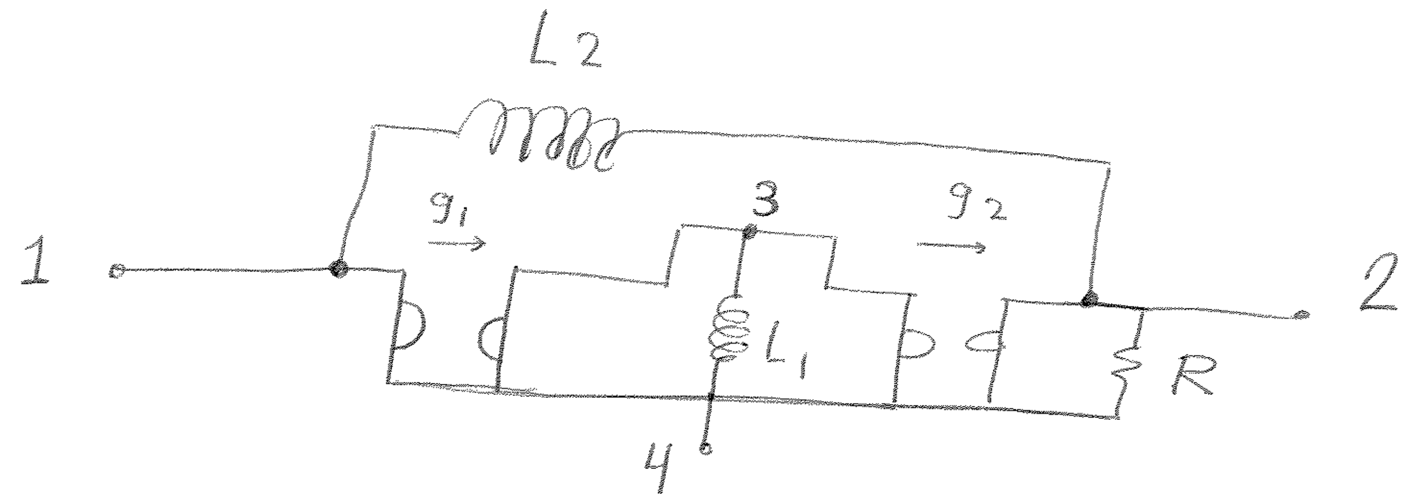
Problem #1



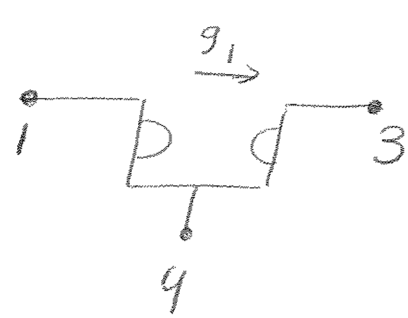
we have 5 elements so we have to write down the admittance matrix for each element then add them up.

$Y_{ind\,g_1}$ & $Y_{ind\,g_2}$ & $Y_{ind\,L_1}$ & $Y_{ind\,L_2}$ & $Y_{ind\,R}$

then

$$Y_{ind(s)} = Y_{ing_1} + Y_{ind\,g_2} + Y_{ind\,L_1} + Y_{inL_2} + Y_{ind\,R}$$

let's first look @ gyrators:



$\Rightarrow$

$$\begin{cases} i_1 = -g_1 \left(V_3 - V_4 \right) \\ i_3 = +g_1 \left(V_1 - V_4 \right) \\ i_4 = +g_1 \left(V_3 - V_1 \right) \end{cases}$$

from KCL: $i_4 = -i_1 - i_3$

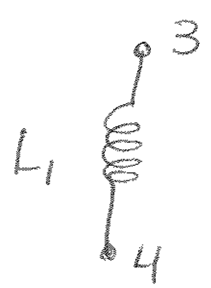
So:

$$Y_{ind_{g_1}} = \begin{bmatrix} 0 & 0 & -g_1 & +g_1 \\ 0 & 0 & 0 & 0 \\ +g_1 & 0 & 0 & -g_1 \\ -g_1 & 0 & +g_1 & 0 \end{bmatrix}$$

Similarly;

$$Y_{ind\, g_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +g_2 & -g_2 \\ 0 & -g_2 & 0 & +g_2 \\ 0 & +g_2 & -g_2 & 0 \end{bmatrix}$$

For inductor, L_1, we have:



we know $V_L = L \dfrac{dI}{dt}$

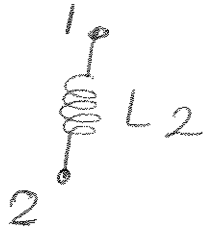
$$\Rightarrow \begin{cases} V_3 - V_4 = Ls\, I_3 \\ V_4 - V_3 = Ls\, I_4 \end{cases}$$

$$\Rightarrow \begin{cases} I_3 = \dfrac{1}{Ls} V_3 - \dfrac{1}{Ls} V_4 \\ I_4 = -\dfrac{1}{Ls} V_3 + \dfrac{1}{Ls} V_4 \end{cases}$$

So:

$$Y_{ind_{L_1}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_1 s} & -\frac{1}{L_1 s} \\ 0 & 0 & -\frac{1}{L_1 s} & +\frac{1}{L_1 s} \end{bmatrix}$$

Similarly;



$$\Rightarrow Y_{ind_{L_2}} = \begin{bmatrix} \frac{1}{L_2 S} & -\frac{1}{L_2 S} & 0 & 0 \\ -\frac{1}{L_2 S} & +\frac{1}{L_2 S} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And, finally, for Resistor:

$$\Rightarrow \begin{cases} i_2 = \frac{V_2}{R} - \frac{V_4}{R} \\ i_4 = \frac{V_4}{R} - \frac{V_2}{R} \end{cases} \Rightarrow Y_{ind_R} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/R & 0 & -1/R \\ 0 & 0 & 0 & 0 \\ 0 & -1/R & 0 & 1/R \end{bmatrix}$$

So:

$$Y_{ind_{(s)}} = \begin{bmatrix} \frac{1}{L_2 S} & -\frac{1}{L_2 S} & -g_1 & +g_1 \; - \\ -\frac{1}{L_2 S} & +\frac{1}{L_2 S} + \frac{1}{R} & +g_2 & -g_2 - \frac{1}{R} \\ +g_1 & -g_2 & \frac{1}{L_1 S} & -g_1 + g_2 - \frac{1}{L} \\ -g_1 & +g_2 - \frac{1}{R} & -\frac{1}{L_1 S} + g_1 - g_2 & +\frac{1}{L_1 S} \; - \end{bmatrix}$$

Part b) Now that we have $Y_{ind(s)}$ as a full 4×4 matrix, we can eliminate internal nodes, i.e; nodes 3 & 4; (4)

lets eliminate node 3 & 4.

<u>First eliminate node "4"</u>:

Ground node $4 \Rightarrow V_4 = 0$ So:

$$Y_{nod(s)} = \begin{bmatrix} \frac{1}{L_2 s} & -\frac{1}{L_2 s} & -g_1 \\ -\frac{1}{L_2 s} & \frac{1}{L_2 s} + \frac{1}{R} & g_2 \\ g_1 & -g_2 & \frac{1}{L_1 s} \end{bmatrix}_{3\times3}$$

Eliminating node "3":

From $Y_{nod(s)}$ we have:

$i_3 = g_1 V_1 - g_2 V_2 + \frac{1}{L_1 s} V_3$; if "i_3" is an external node; then $i_3 = 0 \Rightarrow$

$V_3 = L_1 g_2 s \, V_2 - L_1 g_1 s \, V_1 \Rightarrow V_3 = -L_1 g_1 s V_1 + L_1 g_2 s V_2$

So: from $Y_{nod(s)}$;

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{L_2 s} & -\frac{1}{L_2 s} \\ -\frac{1}{L_2 s} & \frac{1}{L_2 s} + \frac{1}{R} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -g_1 \cdot v_3 \\ +g_2 \cdot v_3 \end{bmatrix}$$

but : $v_3 = \begin{bmatrix} -L_1 g_1 s & L_1 g_2 s \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

So :

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{L_2 s} & -\frac{1}{L_2 s} \\ -\frac{1}{L_2 s} & \frac{1}{L_2 s} + \frac{1}{R} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -g_1 \\ +g_2 \end{bmatrix} \begin{bmatrix} -L_1 g_1 s & L_1 g_2 s \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$\Rightarrow$

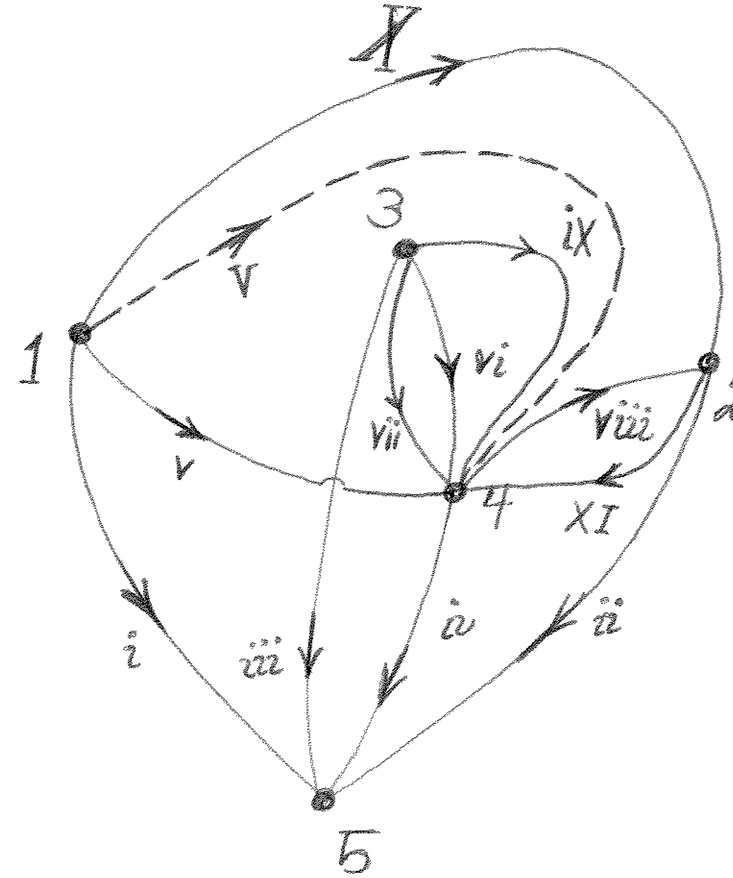
$$Y_{(s)} = \begin{bmatrix} \frac{1}{L_2 s} + L_1 g_1^2 s & -\frac{1}{L_2 s} - L_1 g_1 g_2 s \\ -\frac{1}{L_2 s} - L_1 g_1 g_2 s & \frac{1}{L_2 s} + \frac{1}{R} + L_1 g_2^2 s \end{bmatrix}$$

$\downarrow$

2-port
admittance matrix

Problem #2.

Part a)



Part b)

it is clear from the graph above, if we simply redraw branch V; then we get a planar graph;

it is shown in graph with dashed_line(Red)

So the graph is now planar!

Part c) we have $\underline{5}$ nodes & $\underline{11}$ branches

thus A_a is $\underline{5} \times \underline{11}$ matrix;

So:

$$A_a = \begin{bmatrix} +1 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & +1 \\ 0 & 0 & +1 & 0 & 0 & +1 & +1 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & -1 & +1 & -1 & 0 & -1 \\ -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Part d)

$$n_T = \det(A_a \cdot A_a^T)$$

Using A_a as above & Matlab then:

$$n_T = 0$$

this is what I got; which I guess is not right, I'll check later