ENEE 610
Professor Newcomb
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The University of Maryland, College Park
College Park, MD

Solutions to Homework#1

By:

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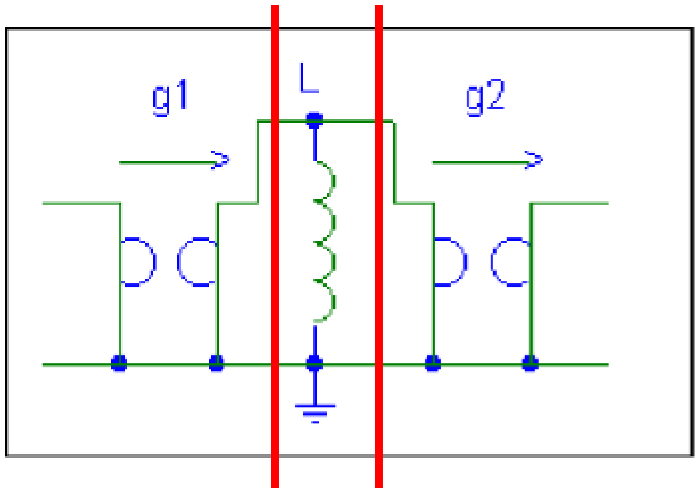
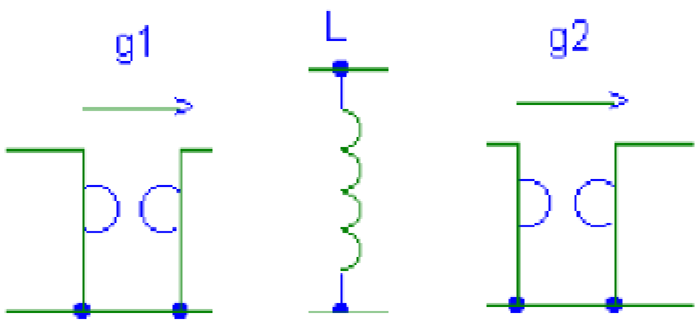
PROBLEM #1

Part a)

We know than the chain matrix of a complex (or a multi component) circuit, can be calculated simply by deriving the chain matrix of each 2-port part of the circuit and then multiplying the cascade of these matrices.

In the following circuit, we have a cascade of three 2-port components:





Therefore, to achieve the "whole" chain matrix, we will simply derive each stage's chain matrix and then multiply them together.

Chain matrix of a gyrator:

Simply from the lecture notes we have:

chain_g1 =

[0, -1/g1]

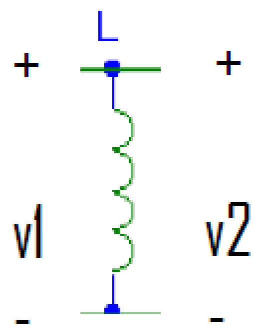
[-g1, 0]

chain_g2 =

[0, -1/g2]

[-g2, 0]

Chain matrix for the inductor:
In order to derive the chain matrix for the inductor, we should write the “terminal relation” for the inductor and then convert it to the ‘s’ or Laplace domain.
We have:



Then we have:
V1 = V2 (KVL)
V1= Ls(I1-I2)
V2= Ls(I2-I1)

Then we have:

chain_L =
[1, 0]
[1/L*s, 1]

Chain for the whole circuit:

Chain = chain_g1 * chain_L * chain_g2

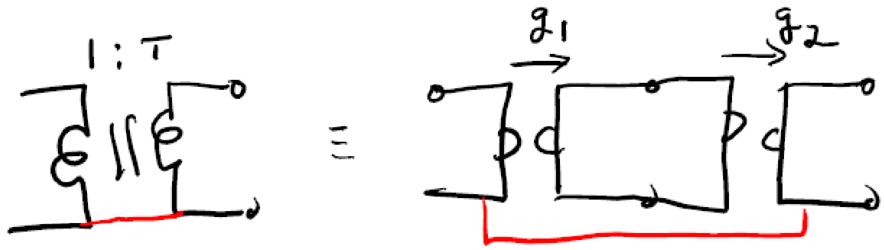
ans =
[1/g1*g2, 1/g1/L*s/g2]
[0, g1/g2]

Let g1/g2 = T

Then

Chain = [1/ g1g2 , Ls/ g1.g2
0 , T]

For the transformer-circuit, we know from lectures that:



$T = g_1/g_2$

So, we can re-write the transformer circuit, and incorporate the capacitors chain as well. We know that the capacitor has the following chain-matrix:

Chain_capacitor = [1 , 1/Cs
0 1];

So the whole circuit has the following chain:

Chain = chain_g1 * chain_g2 * chain_capacitor

ans =

[1/g1*g2, 1/g1*g2/C*s]
[0, g1/g2]

Now if we compare the two circuits we see that the port behavior is the same:

for this chain

```
chain_g1  *  chain_g2  * chain_capacitor
```

the port behavior is

```
[      1/g1*g2, 1/g1*g2/C*s]
[            0,       g1/g2]
```

which is exactly the same as this chain

```
chain_g1  *  chain_L  * chain_g2
```

and its behavior

```
[      1/g1*g2, 1/g1/L*s/g2]
[            0,       g1/g2]
```

Part b) **NEED TO BE CORRECTED**

IF BY EQUALITY AT PORTS WE MEAN THE TWO CIRCUITS BEHAVE THE SAME:

To achieve equality at the ports we should have the previous two matrices be equal. Well, the first assumption is that:

T= g1/g2

Then we have to worry about the only non-matching entry in the matrices above; namely, we should have:

1/g1*g2/C*s = 1/g1/L*s/g2

Which can be simplified to:

Cs / (g1.g2) = Ls / (g1.g2)

Thus to achieve equality at the ports we should have:

T = g1/g2

And

C = L

IF BY EQUALITY AT PORTS WE MEAN THAT THE INPUT AND OUTPUT PORTS ARE REVERSIBLE FOR EACH CIRCUIT THEN WE HAVE:

Now, in order to achieve equality at ports we should have:

I1 = I2 for the 2-port device
V1=V2 again, for the 2-port device

So we can force the "chain matrix" to the following matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Part c)

Knowing the chain-matrix, we can re-write the system-of-equations for the impedance matrix.

V1 = A. V2 – B. I2 eq.1
I1 = C. V2 – D. I2 eq.2

Where [A, B ; C , D] is the chain matrix.
Now, we are looking for

V1 = a. I1 + b. I2
V2 = c. I1 + d. I2

Where [a , b ; c, d] is the impedance matrix.

from eq.2 we have:
V2 = (1/C) . I1 + (D/C) I2

Sub. This for V2 in eq1:
V1 = (A/C) I1 + (AD/C) I2 – B. I2
Or
V1 = (A/C) I1 + [(AD/C) – B] . I2

Thus:

a= A/C **and** **b= (AD/C) – B**

we had:
V2 = (1/C) . I1 + (D/C) I2

Thus:

c= 1/C **and** **d= D/C**

So, the impedance matrix is [a, b; c. d] where a, b, c, and d are given above.

Similarly, one can fine the admittance matrix, Y. For this we should re-write the chain matrix or simply find the inverse of the impedance matrix. One can find that:

```
>> syms A
>> syms B
>> syms C
>> syms D
>> a=A/C;
>> b=((A*D)/C)-B;
>> c=1/C;
>> d=D/C;
>> z=[a b;c d];
>> z

z =

[     A/C, A*D/C-B]
[     1/C,     D/C]
```

```
>> inv(z)

ans =

[         D/B, -(A*D-B*C)/B]
[        -1/B,          A/B]
```

So the admittance matrix is given by

```
[         D/B,                -(A*D-B*C)/B]
[        -1/B,                 A/B]
```

END OF PROBLEM 1
ENEE 610 HW#1
The solution is provided by:
SAEED ESMAILI SARDARI
September 10, 2007

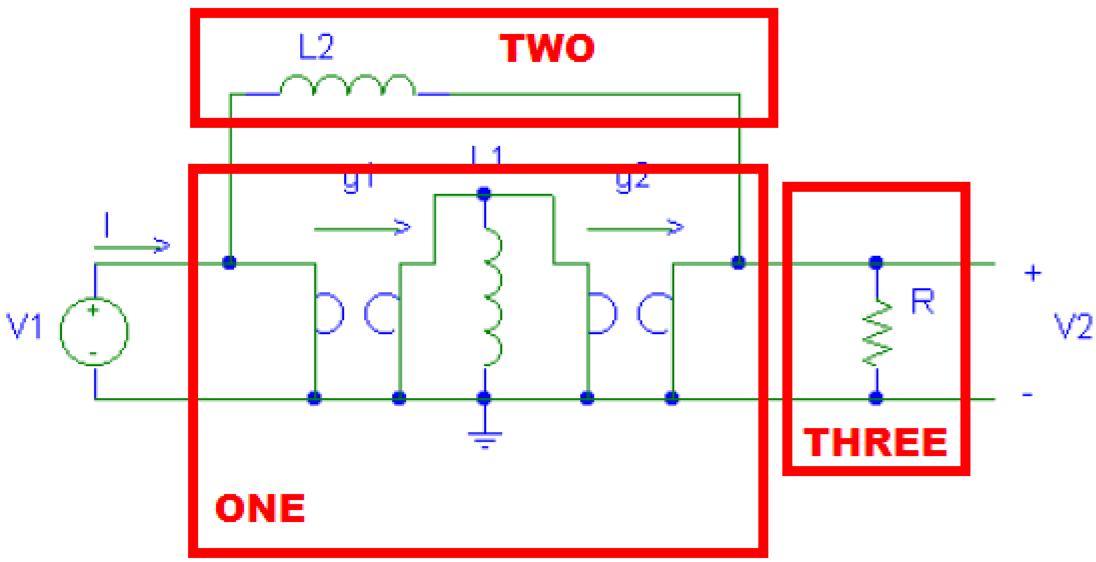
PROBLEM#2

Part a)

Before we start the calculations, lets note that:

In order to find the admittance (or impedance) matrix of a multi-stage circuit, one can find the matrices of individual stages and then ADD the matrices. Note that for chain matrices we MULTIPLY the cascade of chain matrices. So for this problem, we will perform the followings:

1. We will find the Y-matrix of the stage one (look at the figure below)
2. We will find the Y-matrix of the stage two (look at the figure below)
3. We ADD the previous two Y-matrices
4. We convert this Y-matrix to chain matrix
5. We will find the CHAIN matrix of the stage three (look below)
6. We multiply the matrices from step 5, and step 4 above.



Note that stage-one has already been studied in problem1. So we already have the admittance matrix.
Stage-two is a simple inductor with the following admittance matrix:

Y_L = [1/Ls   -1/Ls
         -1/Ls   1/Ls]

So the admittance matrix of the stage-one (ADDED) and stage-two is:

Y_1 = [D/B, -(A*D-B*C)/B]   from problem1.
         [-1/B,   A/B]

Y_2= [1/L2*s, -1/(L2*s);   for the inductor
-1/L2*s, 1/L2*s];

Y= Y_1 + Y_2   two-stage admittace

Y =

[D/B+1/L2*s, -(A*D-B*C)/B-1/L2/s]
[-1/B-1/L2*s, A/B+1/L2*s]

Now we have the following options:

1. Converting this Y to a chain matrix and multiply with chain of ‘R’
2. Finding the Y_for ‘R’ and adding the two-stage Y with Y_R and then converting the Y_total to the chain_total

Let’s follow the first approach. I will find the chain matrix for the Y matrix calculated above.

Y =

[D/B+1/L2*s, -(A*D-B*C)/B-1/L2/s]
[-1/B-1/L2*s, A/B+1/L2*s]

Let:

Y=
[y11 y12
y21 y22]

Where yij’s are:

y11 = D/B+1/L2*s
y12 = -(A*D-B*C)/B-1/L2/s
y21 = -1/B-1/L2*s
y22 = A/B+1/L2*s

We have:

i1 = y11. v1 + y12. v2   eq.3
i2 = y21. v1 + y22. v2   eq.4

To get the chain matrix, we are looking for **A,B,C,** and **D** such that:

v1 = **A**. v2 –**B**. i2
i1 = **C**. v2 –**D**. i2

from eq.4 we have:

$$v1 = (-y22/y21).\ v2 + (1/y21)\ i2$$

and thus:

A = (-y22/y21)
B = -(1/y21)

from eq.3 we have:

$$i1 = y11.\ v1 + y12.\ v2$$

substituting for v1 from eq.4 we have:

$$i1 = y11.\ [\ (-y22/y21).\ v2 + (1/y21)\ i2\ ] + y12.\ v2$$

After simplifying we have:

$$i1 = [\ (-y11.y22/y21) + y12\ ].\ v2 + [\ y11/y21\ ].\ i2$$

Thus:

C = (-y11.y22/y21) + y12
D = - y11/y21

And the chain matrix for the parallel resistor (stage-three) is given by:

$$\text{Chain\_R} = \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}$$

So

The chain matrix for the whole circuit is given by:

Chain_total =

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}$$

Therefore,

Chain_total = [**A** + (**B**/R)   **B**
C + (**D**/R)   **D**]

where **A, B, C,** and **D** are given as:

A = (-y22/y21)
B = -(1/y21)
C = (-y11.y22/y21) + y12
D = - y11/y21

and

y11 = D/B+1/L2*s
y12 = -(A*D-B*C)/B-1/L2/s
y21 = -1/B-1/L2*s
y22 = A/B+1/L2*s

and

A, B, C, and D are the entries of the chain-matrix from problem1.

Note: One may substitute for the parameters above and derive the chain_total matrix as a function of merely, R, L1, L2, and g1, and g2.

Part c) Simulation