EE 610
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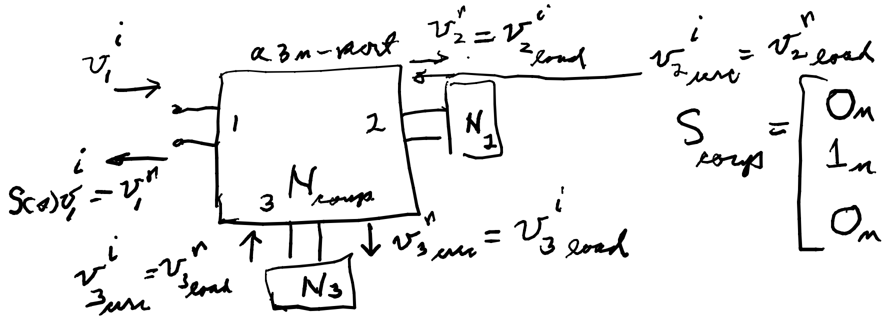


$$S_{circ} = \begin{bmatrix} 0_m & 0_m & 1_m \\ 1_m & 0_m & 0_m \\ 0_m & 1_m & 0_m \end{bmatrix} = 3m\text{-port circulator}$$

N_2 & N_3 described by S_2 & S_3

$$V_{2\,circ}^r = V_{1\,circ}^i \qquad V_{2\,load}^r = S_2 V_{2\,load}^i = S_2 V_{1\,circ}^i$$

$$V_{3\,circ}^r = V_{2\,circ}^i = V_{2\,load}^r = S_2 V_1^i$$

$$= V_{3\,load}^i \Rightarrow V_{3\,load}^r = S_3 \cdot V_{3\,load}^i = S_3\left[S_2 V_{1\,circ}^i\right]$$

$$V_{3\,circ}^i = V_{1\,circ}^r$$

$S(s) = S_3 \cdot S_2$ = input S to loaded circulator

$$S_{circ}^T \cdot S_{circ} = \begin{bmatrix} 0_m & 1_m & 0_m \\ 0_m & 0_m & 1_m \\ 1_m & 0_m & 0_m \end{bmatrix}\begin{bmatrix} 0_m & 0_m & 1_m \\ 1_m & 0_m & 0_m \\ 0_m & 1_m & 0_m \end{bmatrix} = \begin{bmatrix} 1_m & 0_m & 0_m \\ 0_m & 1_m & 0_m \\ 0_m & 0_m & 1_m \end{bmatrix} \Rightarrow \text{lossless}$$

here see the product of two bounded real matrices is still bounded real [note product of positive real need not be positive real $\Rightarrow$ Ex: $s \cdot s = s^2$]

Ex: $n = 1$ circulator: $S_{circ} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$; $Y_{circ} = (\underset{3}{1_n} - S_{circ})(\underset{3}{1_n} + S_{circ})^{-1}$

$1_3 + S_{circ} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$; det = 2, $\Delta_{11} = 1$, $\Delta_{21} = +1$, $\Delta_{31} = -1$

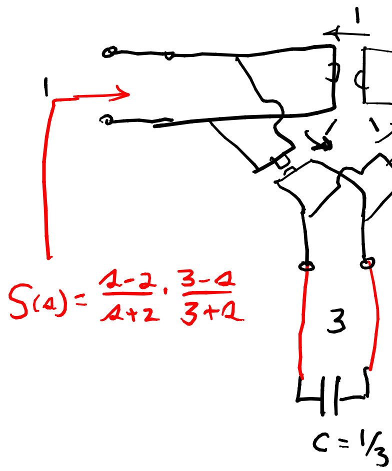
$\Delta_{12} = -1$, $\Delta_{22} = 1$, $\Delta_{32} = +1$

$\Delta_{13} = 1$, $\Delta_{23} = -1$, $\Delta_{33} = 1$

$$(1_3 + S_{circ})^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$Y_{circ} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & +2 \\ +2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$



$S(s) = \frac{s-2}{s+2} \cdot \frac{3-s}{3+s}$

Ex: $S(s) = \frac{s-2}{s+2} \cdot \frac{-s+3}{s+3}$

$S_2(s) = \frac{s-2}{s+2}$; $Y_2 = \frac{1-S_2}{1+S_2} = \frac{s+2-s+2}{s+2+s-2}$

$= \frac{4}{2s} = \frac{1}{\frac{1}{2}s}$

$\Rightarrow$  $L = 1/2$

$S_3(s) = \frac{-s+3}{s+3}$; $Y_3 = \frac{1-S_3}{1+S_3}$

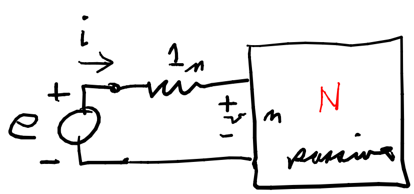
$= \frac{+s+3+s-3}{s+3-s+3}$

$Y_3 = \frac{2s}{6} = \frac{1}{3}s$

$\Rightarrow$  $C = \frac{1}{3}$

The $\mathcal{L}_2$ property of passive N

$x(t) \longrightarrow \int_{-\infty}^{\infty} x^{T*}(t)\, x(t)\, dt$ if finite call $x(t)$ an $\mathcal{L}_2$ vector



$e \in \mathcal{L}_2$; $\int_{-\infty}^{\infty} e^{T*}(t)\, e(t)\, dt$ is ≥ 0 & finite

(≥ 0 as sum of $|\cdot|^2$)

$\cancel{2v}^{\,vi} = e = v + i$

$$\int_{-\infty}^{\infty} \|e(t)\|^2 dt = \int_{-\infty}^{\infty} e^{T*}(t)\, e(t)\, dt$$

$$= \int_{-\infty}^{\infty} (v(t)+i(t))^{T*} (v(t)+i(t))\, dt$$

$$= \int_{-\infty}^{\infty} 4 v^{iT*} v^{i}\, dt = \int_{-\infty}^{\infty} \left\{ \|v(t)\|^2 + \|i(t)\|^2 + 2\,\mathrm{Re}\left(v^{T*}(t)\, i(t)\right) \right\} dt$$

≥ 0 & finite for $e \in \mathcal{L}_2$

if passive $\int_{-\infty}^{\infty} 2\,\mathrm{Re}\left(v^{T*}(t)\, i(t)\right) dt \geq 0$

$\Rightarrow$ every term is finite & $\geq 0 \Rightarrow v \in \mathcal{L}_2,\ i \in \mathcal{L}_2$

Look at $2v^r = v - i$

$$\int_{-\infty}^{\infty} 4\, v^{rT*}(t)\, v^{r}(t)\, dt = \int_{-\infty}^{\infty} (v-i)^{T*}(v-i)\, dt = \int_{-\infty}^{\infty} \|v\|^2 + \|i\|^2 - 2\,\Re\, v^{T*} i \; dt$$

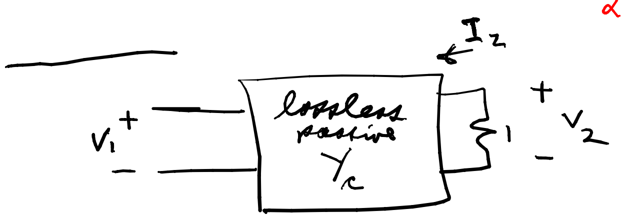
If subtract the $\int 4\|v^r\|^2$ from $\int 4\|v^i\|^2$ we get $\int 4 \Re\, v^{T*} i\, dt \geq 0$

$$\int_{-\infty}^{\infty} \|v^i\|^2 dt - \int_{-\infty}^{\infty} \|v^r\|^2 dt \geq 0$$

$$\int_{-\infty}^{\infty} \{ \|v^i\|^2 - \|v^r\|^2 \} dt \geq 0$$ $\Rightarrow v^r \in \mathcal{L}_2$ if $v^i \in \mathcal{L}_2$ & N passive

$$\Rightarrow \int_{-\infty}^{\infty} \{ 1 \cdot \|v^i\|^2 - \|S(t) * v^i\|^2 \} dt = \underbrace{\int_{-\infty}^{\infty} \{ 1 - \|S(t)\|^2 \} \underbrace{\|v^i\|^2}_{\geq 0} dt}_{\geq 0}$$

$1 - \|S(t)\|^2 \geq 0 \Rightarrow$ S as an operator mapping $\mathcal{L}_2$ into $\mathcal{L}_2$ is bounded in norm by 1 if N is passive



$$-V_2 = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

If y_{22} = reactance function

$$-V_2 = y_{21} V_1 + y_{22} V_2$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{-y_{21}}{1 + y_{22}} = \frac{n_{21}/D}{1 + \frac{n_{22}}{D}} = \frac{n_{21}}{\underset{\parallel\, d_{22}}{n_{22} + D}}$$

$\Rightarrow P(s) = n_{22}(s) + D(s) =$ Hurwitz polynomial $= \mathcal{E}v P(s) + \mathcal{O}d P(s)$

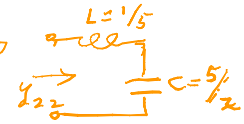
$\frac{n_{22}}{D} = y_{22} = \frac{\text{even}}{\text{odd}}$ or $\frac{\text{odd}}{\text{even}} \Rightarrow \frac{\mathcal{E}v P}{\mathcal{O}d P} = y_{22} \Rightarrow$ can synthesize by Cauer, Foster to get zeros of n_{21}

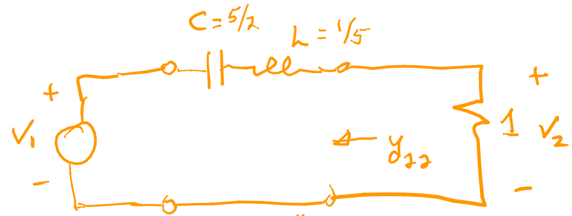
added example:

$$\frac{V_2}{V_1} = \frac{ks}{s^2 + 5s + 2} = \frac{ks/(s^2+2)}{1 + 5s/(s^2+2)} \Rightarrow y_{22} = \frac{5s}{s^2+2},\ y_{21} = \frac{-ks}{s^2+2}$$

synthesize y_{22} by a ladder (= Cauer) such that there is one zero of transmission at $s=0$ & one at $s=\infty$ by series arm open or shunt arm short

$$y_{22} = \frac{1}{\frac{s}{5} + \frac{2}{5s}} \Rightarrow$$





find k for y_{21} by analysis

$$\frac{V_2}{V_1} = \frac{1}{1 + \left(\frac{1}{5}s + \frac{1}{\frac{5}{2}s}\right)} = \frac{5s}{5s + s^2 + 2}$$

zeros of transmission = zeros of series arm } 0 at 0 0 at ∞

$\therefore$ k = 5

here $y_{11} = y_{22} = -y_{12} = -y_{21}$ as a simple & special case