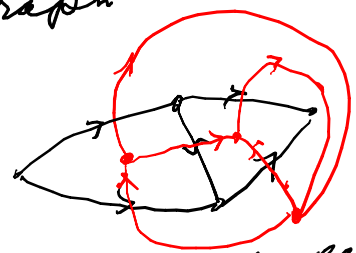
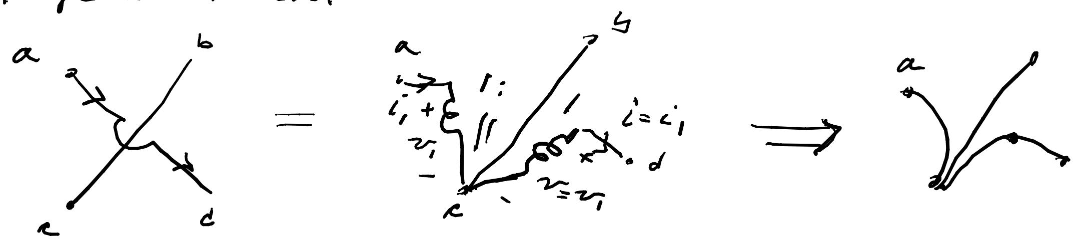
EE 610
10/22/07

Ex of dual graph



If non-planar can't do this construction

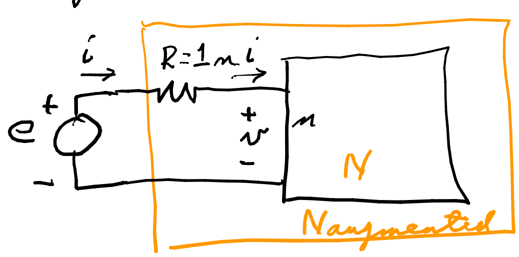


now no crossing wires and all currents & voltages are preserved

Scattering matrix

incident $\Rightarrow 2v^i = v + z_0 i$; $z_0 = R = 1_n$

reflected $\Rightarrow 2v^r = v - z_0 i$



$2v = 2v^i + 2v^r \Rightarrow v = v^i + v^r$

$e = v + Ri = v + i = 2v^i$

$i = Y_a \cdot e$

$v = e - i = e - Y_a e = (1_n - Y_a)e$

$v - i = e - 2i$
$= e - 2Y_a e$
$= (1_n - 2Y_a)e$

$v^i = \frac{v+i}{2} = \frac{e}{2}$

$v^r = \frac{v-i}{2} = (1_n - 2Y_a)\frac{e}{2}$

$= (1_n - 2Y_a)v^i \Rightarrow v^r = S(s) \cdot v^i$

$S(s) = 1_n - 2Y_a(s)$

also if $i = Yv$; $e = v + Yv = 2v^i$

$2v^r = v - i = v - Yv$ $= v^r = (1_n - Y)\frac{v}{2}$

$v^i = (1_n + Y)v/2$

$$\frac{v}{2} = (1_n + Y)^{-1} v^i \Rightarrow v^r = (1_n - Y)\frac{v}{2} = (1_n - Y)(1_n + Y)^{-1} \cdot v^i$$

$$S(s) = (1_n - Y(s))(1_n + Y(s))^{-1}$$; if N is passive $(1_n + Y(s))^{-1}$ exists in $\mathrm{Re}\, s > 0$ if $Y(s)$ exists

$$(1_n - Y)(1_n + Y)^{-1} = (1_n + Y)^{-1}(1_n - Y)$$ $\times (1_n + Y)$ on right & left

$$(1_n + Y)(1_n - Y) = (1_n - Y)(1_n + Y)$$
$$= 1_n - Y^2 \qquad\qquad = 1_n - Y^2$$

$$P(j\omega) = \mathrm{Re}(V^{*T} I) = \frac{V^{*T} I + V^T I^*}{2} = \frac{V^{*T} I + I^{*T} V}{2} \geq 0 \text{ if passive}$$

$V = V^i + V^r$
$I = V^i - V^r$

$$= \frac{(V^i + V^r)^{*T}(V^i - V^r) + (V^i - V^r)^{*T}(V^i + V^r)}{2}$$

$$= V^{i*T} V^i - \underline{\underline{V^{i*T} V^r}} + \underline{\underline{V^{r*T} V^i}} - V^{r*T} V^r = V^{i*T} V^i - V^{r*T} V^r$$

conjugates

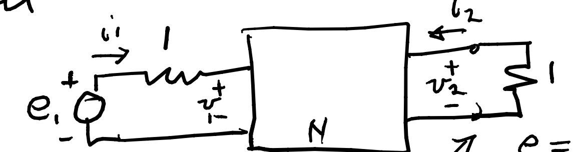
$$P(j\omega) = V^{i*T} V^i - V^{i*T} S^{T*}(j\omega) S(j\omega) V^i = V^{i*T}\left[1_n - S^{T*}(j\omega) S(j\omega)\right] V^i \geq 0$$ if passive

$\therefore$ for a passive circuit $S(s)$ is bounded-real

1) $S(s)$ is real for $s = \sigma > 0$
2) $S(s)$ is analytic in $\sigma > 0$
3) $1_n - S^{T*}(s) S(s)$ is positive semi-definite in $\sigma > 0$

Can not have poles on $j\omega$ axis as $|S|^2 \leq 1$ for $s = j\omega$

For a 2-port



$e_2 \equiv 0$ $\Downarrow$ $v_2^i = 0$

$$v^i = \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} = \begin{bmatrix} e_1/2 \\ 0 \end{bmatrix}$$

$$v^r = \begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} (v_1 - i_1)/2 \\ \frac{v_2 - i_2}{2} \end{bmatrix} = \begin{bmatrix} v_1^r \\ 2v_2/2 \end{bmatrix}$$

$$S_{21} = \frac{v_2^r}{v_1^i} = \frac{v_2}{e_1/2} = 2\left(\frac{v_2}{e_1}\right)$$

$v_2 = -i_2$
$v_2^r = v_2$

$$S_{11} = \frac{v_1^r}{v_1^i} = \left.\frac{(v_1 - i_1)/2}{(v_1 + i_1)/2}\right|_{v_2^i = 0} = \text{reflection coefficient}$$

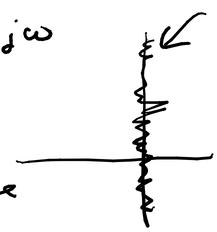
if $S_{11} = 0$ input to N when loaded looks like a unit resistor ≡ match

If N is lossless $P(j\omega) = 0 \Rightarrow 1_n - S^{T*}(j\omega)S(j\omega) = 0_n$

$$= 1_n - S^T(-j\omega)S(j\omega)$$

$$= 1_n - S^T(-s)S(s)\Big|_{s=j\omega} = 0_n$$

$= 0$

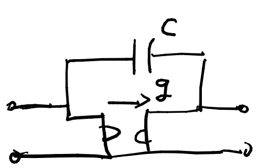


if $S(s)$ is rational this is for every ω, $-\infty < \omega < \infty$

if $S(s)$ is rational and N is lossless & passive then $1_n - S^T(-s)S(s) = 0_n$ for all s

$$\Rightarrow 1_n = S^T(-s)S(s) \text{ or } S^{-1}(s) \text{ exists } \& \ S^{-1}(s) = S^T(-s)$$

Ex:



$$Y(s) = \begin{bmatrix} sC & -sC-g \\ -sC+g & sC \end{bmatrix}$$

$$S(s) = (1_2 - Y)(1_2 + Y)^{-1}$$

$$1_2 + Y = \begin{bmatrix} 1+sC & -sC-g \\ -sC+g & 1+sC \end{bmatrix}; \quad (1+sC)^2 - (-sC+g)(-sC-g) = \Delta Y$$

$$= 1 + 2sC + \cancel{(sC)^2} - \cancel{(sC)^2} + g^2 = 1 + 2sC + g^2$$

$$= (1+g^2) + 2sC$$

$$S(s) = \frac{1}{(1+g^2) + 2sC}\begin{bmatrix} 1-sC & sC+g \\ sC-g & 1-sC \end{bmatrix}\begin{bmatrix} 1+sC & sC+g \\ sC-g & 1+sC \end{bmatrix}$$

poles at $s = -\dfrac{(1+g^2)}{2C}$ in LHP

$$m_{11} = (1-sC)(1+sC) + (sC+g)(sC-g) = 1 - \cancel{(sC)^2} + \cancel{(sC)^2} - g^2$$

$$m_{12} = (1-sC)(sC+g) + (sC+g)(1+sC) = 2(sC+g)$$

$$m_{21} = (sC-g)(1+sC) + (1-sC)(sC-g) = 2(sC-g)$$

$$m_{22} = (sC-g)(sC+g) + (1+sC)(1-sC) = m_{11}$$

$$S(\alpha) = \frac{1}{\alpha c + (1+g^2)} \begin{bmatrix} 1-g^2 & 2(\alpha c+g) \\ 2(\alpha c - g) & 1-g^2 \end{bmatrix}$$

check $S^T(-\alpha)\, S(\alpha)$ should $= 1_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\frac{1}{-2\alpha c + (1+g^2)} \begin{bmatrix} 1-g^2 & 2(-\alpha c - g) \\ 2(-\alpha c + g) & 1-g^2 \end{bmatrix} \frac{1}{2(\alpha c) + (1+g^2)} \begin{bmatrix} 1-g^2 & 2(\alpha c+g) \\ 2(\alpha c - g) & 1-g^2 \end{bmatrix}$$

$$= \frac{1}{(1+g^2)^2 - 4(\alpha c)^2} \begin{bmatrix} (1-g^2)^2 - 4((\alpha c)^2 - g^2) & \underbrace{(1-g^2)2(\alpha c+g) - 2(\alpha c+g)(1-g^2)}_{=0} \\ \underbrace{2(-\alpha c+g)(1-g^2) + (1-g^2)2(\alpha c - g)}_{=0} & 4(-\alpha c+g)(\alpha+g) + (1-g^2)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

here $(1-g^2)^2 - 4(\alpha c)^2 + 4g^2$

$= -4(\alpha c)^2 + 1 - 2g^2 + g^4 + 4g^2$

$= -4(\alpha c)^2 + (1+g^2)^2$