EE610
10/17/07

$$(s\mathbb{1}_2 - A)^{-1} = \frac{1}{s^2 - 5s + 7}\begin{bmatrix} s-4 & -1 \\ 3 & s-1 \end{bmatrix} = \frac{1 \begin{bmatrix} s-4 & -1 \\ 3 & s-1 \end{bmatrix}}{\left(s - \frac{1}{2}(5 - j\sqrt{3})\right)\left(s - \frac{1}{2}(5 + j\sqrt{3})\right)}$$

$$= \frac{1}{\left(s - \frac{1}{2}(5 + j\sqrt{3})\right)}\begin{bmatrix} \quad \end{bmatrix} + \frac{1}{\left(s - \frac{1}{2}(5 - j\sqrt{3})\right)}\begin{bmatrix} \quad \end{bmatrix}$$

$$\left.\frac{s-4}{s - \frac{1}{2}(5 - j\sqrt{3})}\right|_{s = +\frac{1}{2}(5 + j\sqrt{3})} = \frac{-3/2 + j\sqrt{3}/2}{2j\sqrt{3}/2} = \frac{1}{2} + \frac{\sqrt{3}}{2}j$$

$$\left.\frac{-1}{s - \frac{1}{2}(5 - j\sqrt{3})}\right|_{s = \frac{1}{2}(5 + j\sqrt{3})} = \frac{-1}{2j\frac{\sqrt{3}}{2}} = (+1/\sqrt{3})j$$

$$\frac{3}{2j\sqrt{3}/2} = -\sqrt{3}j, \quad \left.\frac{s-1}{2j\frac{\sqrt{3}}{2}}\right|_{s = \frac{1}{2}(5 + j\sqrt{3})} = \frac{(3 + j\sqrt{3})/2}{2j\sqrt{3}/2} = \frac{1}{2} - j\sqrt{3}/2$$

$$(s\mathbb{1}_2 - A)^{-1} = \frac{1}{s - \frac{1}{2}(5 + j\sqrt{3})}\begin{bmatrix} \frac{1}{2} + j\frac{\sqrt{3}}{2} & j/\sqrt{3} \\ -\sqrt{3}j & \frac{1}{2} - j\sqrt{3}/2 \end{bmatrix} + \frac{1}{s - \frac{1}{2}(5 - j\sqrt{3})}\begin{bmatrix} \frac{1}{2} - j\frac{\sqrt{3}}{2} & -j/\sqrt{3} \\ \sqrt{3}j & \frac{1}{2} + j\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\mathcal{L}^{-1}\left[(s\mathbb{1}_2 - A)^{-1}\right] = \begin{bmatrix} \frac{1}{2} + j\frac{\sqrt{3}}{2} & j\sqrt{3} \\ -\sqrt{3}j & \frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix} e^{\frac{1}{2}(5 + j\sqrt{3})t} + \begin{bmatrix} \frac{1}{2} - j\frac{\sqrt{3}}{2} & -j/\sqrt{3} \\ \sqrt{3}j & \frac{1}{2} + j\frac{\sqrt{3}}{2} \end{bmatrix} e^{\frac{1}{2}(5 - j\sqrt{3})t}$$

$$= e^{\frac{1}{2}5t}\left\{\begin{bmatrix} \frac{1}{2}+j\frac{\sqrt{3}}{2} & j\sqrt{3} \\ -\sqrt{3}j & \frac{1}{2}-j\frac{\sqrt{3}}{2} \end{bmatrix}\left(\cos\frac{\sqrt{3}}{2}t + j\sin\frac{\sqrt{3}}{2}t\right) + \text{conjugate}\right\} = e^{At} = e^{\begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}t}$$

$$= A^0 + At + \frac{1}{2}(A)^2 t^2 + \cdots$$

Design technique $\quad T(s) = Y(s) =$ admittance matrix, $n \times n$

$u = v(t) = n$-vector of inputs

$y = i(t) = n$-vector of currents at the same n ports

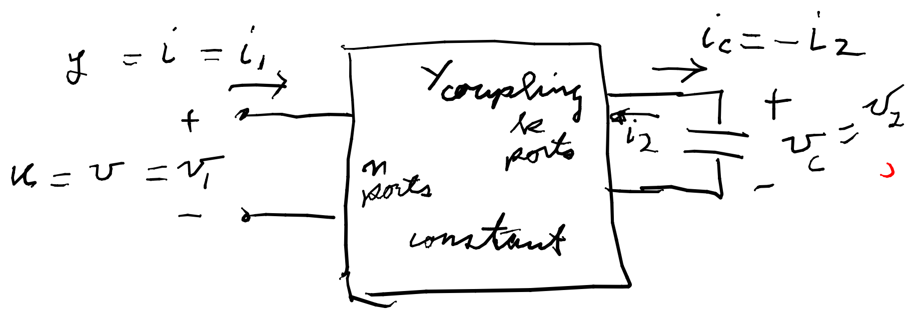
assume have state variable equations

$$i_c = \underset{\text{cap}}{C}\dot{v}_c \quad \Rightarrow \dot{x} = Ax + Bu = Ax + Bv$$

$$i = y = Cx + Du = Cx + Dv$$

let $x = v_c$ = capacitor voltages

$\dim x = k$



$C_{ap} = 1_k$

$$\begin{array}{r} y \\ -\dot{x} \quad -i_c \end{array} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underbrace{\begin{bmatrix} D & C \\ -B & -A \end{bmatrix}}_{} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{array}{l} = u \\ = x \end{array}$$

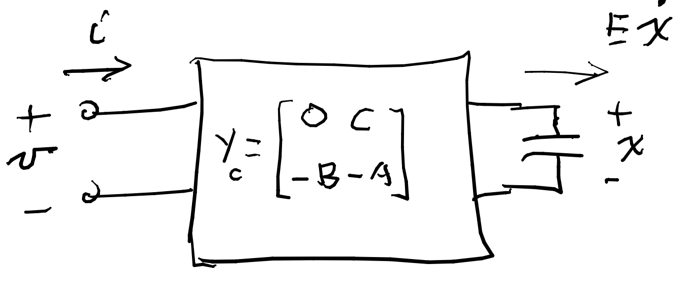
$Y_{coupling}$ = purely "resistive"

can make with

VCCS $\Rightarrow$ G of spice

If the equations are

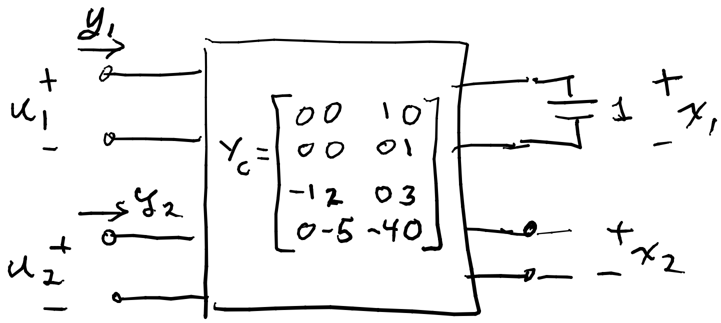
$$E\dot{x} = Ax + Bu$$
$$y = Cx$$



so $C_{ap} = E$

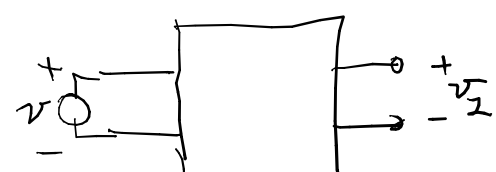
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 0 & -3 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

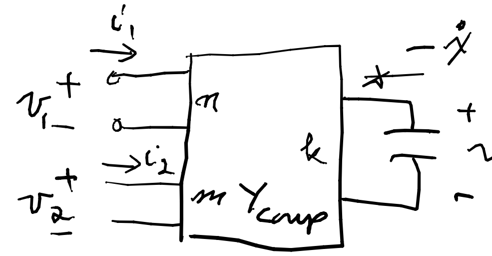
$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$$



if $y = v_2$, $u = v_1$

$$\dot{x} = Ax + Bv_1$$
$$y = v_2 = Cx + Dv_1$$



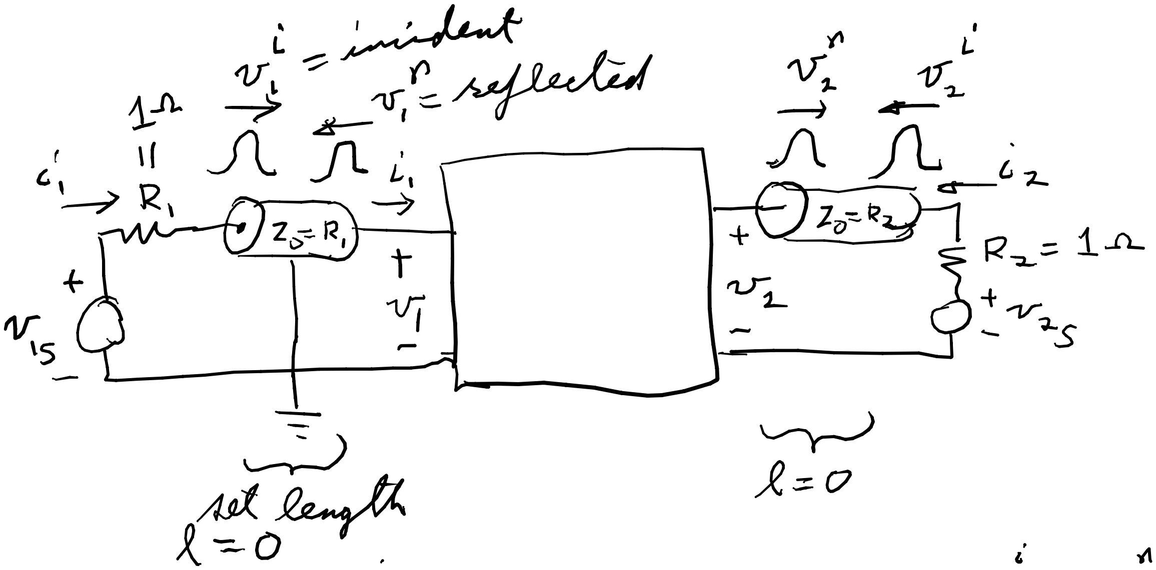


$v_3 = x$, $C_{ap} = 1_k$

$$\begin{array}{r} \text{don't care} \\ 0 \\ -\dot{x} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \bullet & \bullet & \bullet \\ -D & 1_m & -C \\ -B & 0 & -A \end{bmatrix}}_{Y_{\text{group}}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \begin{array}{l} \\ \\ x \end{array}$$

$\bullet$ = free to choose

Y_{group}; choose $\bullet$ to make it "nice"



$$\left.\begin{aligned} 2v_1^i &= v_1 + Z_0 i_1 = v_1 + i_1 \\ 2v_1^r &= v_1 - Z_0 i = v_1 - i_1 \end{aligned}\right\} \quad \begin{aligned} 2v_1^i + 2v_1^r &= 2v_1 \\ 2v_1^i - 2v_1^r &= 2i_1 \end{aligned}$$

give

$$\left.\begin{aligned} 2v^i &= v + i \\ 2v^r &= v - i \end{aligned}\right\} \quad \begin{aligned} v &= v^i + v^r \\ i &= v^i - v^r \end{aligned}$$

$$V^r(s) = S(s)\, V^i(s) \quad ; \quad S(s) = \text{scattering matrix}$$

$$\frac{1}{2}\left[V - I\right] = S\,\frac{1}{2}\left[V + I\right]$$

$$\frac{1}{2}V - S\frac{1}{2}V = \frac{1}{2}I + \frac{1}{2}SI = \frac{(1+S)I}{2}$$

$$= \frac{1}{2}(1 - S)V$$

$$I = (1+S)^{-1}(1-S)V$$

$$Y = (1+S)^{-1}(1-S)$$

$$\Rightarrow \quad (1+S)Y = 1-S \Rightarrow SY+S = 1-Y$$

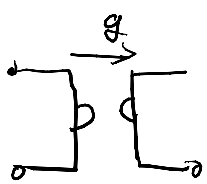
$$= S(Y+1) =$$

$$S = (1-Y)(1+Y)^{-1}$$

Ex: c  $y(s) = cs$; $S(s) = \dfrac{1-cs}{1+cs}$

$$S(-s) = \frac{1}{S(s)} \Rightarrow S(-s)S(s) = 1$$

$$1 - S(-s)S(s) = 0$$



$$Y(s) = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & g \\ -g & 1 \end{bmatrix}\begin{bmatrix} 1 & -g \\ g & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & g \\ -g & 1 \end{bmatrix}\frac{1}{1+g^2}\begin{bmatrix} 1 & g \\ -g & 1 \end{bmatrix} = \begin{bmatrix} 1-g^2 & 2g \\ -2g & 1-g^2 \end{bmatrix}\frac{1}{1+g^2}$$

$$S^T S = \frac{1}{(1+g^2)^2}\begin{bmatrix} 1-g^2 & -2g \\ 2g & 1-g^2 \end{bmatrix}\begin{bmatrix} 1-g^2 & 2g \\ -2g & 1-g^2 \end{bmatrix}$$

$$= \frac{1}{(1+g^2)^2}\begin{bmatrix} (1-g^2)^2+4g^2 & 0 \\ 0 & (1-g^2)^2+4g^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$