(color of 10/12/07 corrections)

EE 610
10/10/07

Homework extended until M 10/15/07

P. J. Richards, "A Special Class of Functions with Positive Real part in Half Plane," Duke Mathematical Journal, Vol. 14 1947, pp. 777-786. (on line via UMD library, E-journals)

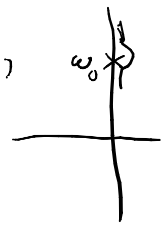
PR Test of p. 339 rational with real coefficients $\Rightarrow y(s) = \dfrac{N(s)}{D(s)}$
N & D polynomial in s

(a) $D(s)$ has no zeroes in $\mathrm{Re}\, s > 0$

(b) poles on $j\omega$ axis are simple, residues > 0

$$\frac{k}{s - j\omega_0} = \frac{|k| e^{j\angle k}}{|s - j\omega_0|\, e^{j\angle(s - j\omega_0)}}$$

$\overset{\shortparallel}{r}$ = radius



ω_0

$$\text{Ex: } \frac{4s}{s^2+3} = \frac{k_1}{(s+j\sqrt{3})} + \frac{k_2 = k_1^* = k_1}{(s - j\sqrt{3})}$$

$$\Downarrow \quad = \frac{2k_1 s}{s^2+3}$$

$\Rightarrow k_1 = 2$

$$y(s) = \frac{4s}{s^2+3}$$

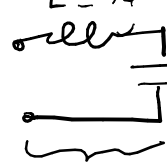
$-\pi/2 < (\angle s - j\omega) < \pi/2$ as $\sigma > 0$

$\Rightarrow \angle k = 0$ as otherwise $\mathrm{Re}(e^{j\angle k - j\angle(s - j\omega)})$ would be negative

$\Downarrow$

k real & positive

$$z(s) = \frac{1}{y(s)} = \frac{s^2+3}{4s} = \frac{1}{4}s + \frac{1}{\frac{4}{3}s} \Rightarrow$$



$L = 1/4$

$C = 4/3$

passive $\Rightarrow y(s)$ is PR

(c) $\mathrm{Re}\, y(j\omega) \geq 0$ for $-\infty \leq \omega \leq \infty \equiv 0 \leq \omega \leq \infty$

$$\text{Ev } y(s)\Big|_{s=j\omega} = \mathrm{Re}\, y(j\omega) = \frac{y(s) + y(-s)}{2}\Big|_{s=j\omega} = \frac{y(j\omega) + y(-j\omega)}{2}$$

$$= \frac{y(j\omega) + y^*(j\omega)}{2}$$

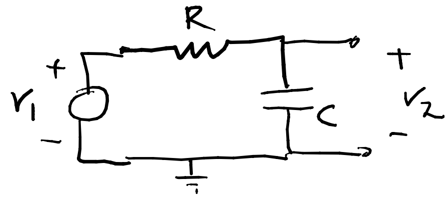
$$= \mathrm{Re}\, y(j\omega)$$

gives a test for PR property

sensitivity of $T(s)$ with respect to a parameter x

$$S_x^{T(s)} = \frac{1}{\frac{T(s)}{x}} \frac{dT(s)}{dx}$$

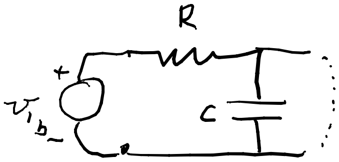
can use the adjoint circuit (p. 406 of Paikari)



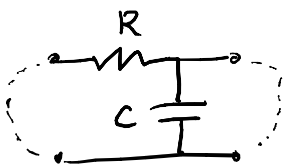
$$T(s) = \frac{V_2}{V_1} = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs+1}$$

$$S_R^T = \frac{R}{\frac{1}{RCs+1}} \cdot \frac{d\left(\frac{1}{RCs+1}\right)}{dR} = \frac{-RCs}{RCs+1}$$

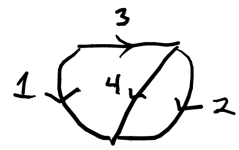
$$\frac{d}{dR}\left(\frac{1}{sCR+1}\right) = \frac{-1}{(RCs+1)^2} \times Cs = -\frac{Cs}{(RCs+1)^2}$$



2-port



adjoint of the 2-port



$v_d = \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}, \; i_d = \begin{bmatrix} i_3 \\ i_4 \end{bmatrix}$

$i_b^T v_b = 0$ use $\Delta i_b, \Delta v_b$

i_b^a, v_b^a

$$\Delta i_b^T v_b^a - i_b^{aT} \Delta v_b = 0 \Rightarrow \Delta i_{b_1} v_{b_1}^a + \Delta i_{b_2} v_{b_2}^a - i_{b_1}^a \Delta v_{b_1} - i_{b_2}^a \Delta v_{b_2}$$
$$+ \Delta i_d^T v_d^a - i_d^{aT} \Delta v_b = 0$$

fix v_{1_b}, C, and adjoint components, Δv_{2_b}

and set $v_{1_b} = 1$ so $T(s) = V_2(s)$

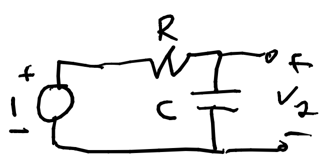
as not interested in Δi_{b_1} set $v_{b_1}^a = 0 \Rightarrow$ short

set $i_{b_2}^a = \pm 1 \Rightarrow$ current source

adjoint

on original $\Delta i_{b_2} = 0$ as an open circuit when measure V_2

$\Delta v_{b_1} = 0$ as $v_{b_1} = 1 = v_{1_b}$



(as $i_{b_2}^a = \pm 1$)

$$\pm i_{b_2}^a \Delta v_{b_2} = \pm \Delta v_{b_2} = \pm \Delta T(s) = \Delta i_d^T v_d^a - i_d^{aT} \Delta v_d$$

$$\left(\Delta\left[i_d = \underset{b\times b}{Y} v_d\right]\right)^T v_d^a - \left(i_d^a = \underset{b\times b}{Y^a} v_d^a\right)^T \Delta v_d$$

$b \times b$ = branch × branch of the 2-port. ($\neq$ 2-porty)

$$= \left(\underset{b\times b}{\Delta Y} \cdot v_d + \underset{b\times b}{Y} \Delta v_d\right)^T v_d^a - v_d^{aT} \underset{b\times b}{Y^{aT}} \cdot \Delta v_d$$

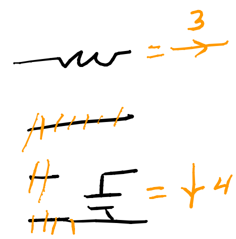
$$= v_d^T \underset{b\times b}{\Delta Y^T} v_d^a + \Delta v_d^T \underset{b\times b}{Y^T} v_d^a - v_d^{aT} \underset{b\times b}{Y^{aT}} \Delta v_d$$

$$\| \quad \Delta v_d^T \underset{b\times b}{Y^a} v_d^a$$

force $= 0 \Rightarrow \Delta v_d^T \left(\underset{b\times b}{Y^T} - \underset{b\times b}{Y^a}\right) v_d^a$

$$\underset{b\times b}{Y^a} = \underset{b\times b}{Y^T}$$

$$\pm \Delta T(a) = v_d^T \left(\underset{b\times b}{\Delta Y^T}\right) v_d^a$$



here $\underset{b\times b}{Y} = \begin{bmatrix} G & \overset{=0}{\cancel{-G}} \\ \overset{=0}{\cancel{-G}} & \cancel{G} + sC \end{bmatrix}$

$$= v_d^{aT} \underset{b\times b}{\Delta Y} v_d$$

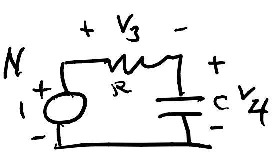
$$\Delta G = \Delta \frac{1}{R} = \frac{-1}{R^2} \Delta R$$

$$\underset{b\times b}{\Delta Y} = \frac{-1}{R^2} \Delta R \begin{bmatrix} 1 & \overset{0}{\cancel{-1}} \\ \underset{0}{\cancel{-1}} & \underset{0}{\cancel{1}} \end{bmatrix}$$

Need $\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = v_d$, $\begin{bmatrix} V_3^a \\ V_4^a \end{bmatrix} = v_d^a$

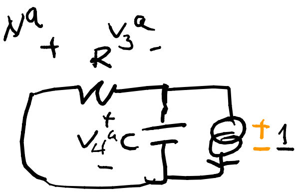
$$\pm \frac{\Delta T(a)}{\Delta R} = \begin{bmatrix} V_3 & V_4 \end{bmatrix} \begin{bmatrix} -1/R^2 & \overset{0}{\cancel{1/R^2}} \\ \overset{0}{\cancel{1/R^2}} & \overset{0}{\cancel{-1/R^2}} \end{bmatrix} \begin{bmatrix} V_3^a \\ V_4^a \end{bmatrix}$$

analyze N



$$V_3 = \frac{R}{R + 1/sC}, \quad V_4 = \frac{1/sC}{R + 1/sC}$$

analyze N^a



here $V_3^a = -V_4^a = z_{RC} \times (\pm 1)$

$$= \frac{\pm 1}{G + sC} \times 1$$

$\therefore$

$$\pm \frac{\Delta T(a)}{\Delta R} = \left(\frac{-1}{R^2}\right) V_3 V_3^a = \frac{-1}{R^2} \cdot \frac{R}{(R + 1/sC)} \cdot \frac{\pm 1}{G + sC} = \frac{\mp sC}{(sCR + 1)^2}$$

shows either sign on $i^a_{b_2}$ gives the same result