$y(s) = \left(\frac{s-1}{s+2}\right)^2$ Re for $s = \sigma > 0$; no pole in $\sigma > 0$, poles at $s = -2$

EE 610
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Re $y(s)$, $s = \sigma + j\omega$; $\sigma > 0$

$\mathrm{Re}(y(s)) = \frac{1}{2}[y(s) + y(s)^*]$, $* \Rightarrow j \to -j$, $j = \sqrt{-1}$

$$2\,\mathrm{Re}(y(s)) = \left(\frac{\sigma + j\omega - 1}{s+2}\right)^2 + \left(\frac{\sigma - j\omega - 1}{s^*+2}\right)^2$$

$$= \frac{(\sigma+j\omega-1)^2(\sigma-j\omega+2)^2 + (\sigma-j\omega-1)^2(\sigma+j\omega+2)^2}{(s+2)(s+2)^*}$$

$$= 2|s+2|^2\,\mathrm{Re}\,y(s) = \left((\sigma+j\omega)(\sigma-j\omega) - \sigma + j\omega + 2\sigma + 2j\omega - 2\right)^2 + \left((\sigma+j\omega)(\sigma-j\omega) - \sigma - j\omega + 2\sigma - 2j\omega - 2\right)^2$$

$$= \left([\sigma^2+\omega^2+1\sigma-2] + j3\omega\right)^2 + \left([\sigma^2+\omega^2+\sigma-2] - j3\omega\right)^2$$

$$= [\sigma^2+\omega^2+\sigma-2]^2 - 9\omega^2 + \cancel{j3\omega[\sigma^2+\omega^2+\sigma-2] - j3\omega[\sigma^2+\omega^2-2]} + [\sigma^2+\omega^2+\sigma-2]^2 - 9\omega^2$$

choose $\sigma = 1$, $\omega = 1$ $\quad 1^2 - 9 < 0 \Rightarrow y(s)$ is not positive real

For this $y(s)$, $\mathrm{Ev}(y(s)) = \frac{1}{2}(y + y_*)$, lower $_*$ $s \to -s$

$$2\,\mathrm{Ev}\,y(s) = \left(\frac{s-1}{s+2}\right)^2 + \left(\frac{-s-1}{-s+2}\right)^2 = \frac{(s-1)^2(-s+2)^2 + (s+1)^2(s+2)^2}{(s+2)(-s+2)}$$

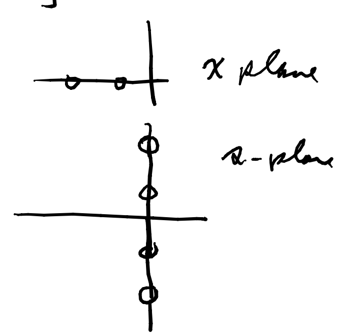
Zeros of $\mathrm{Ev}\,y(s)$ are zeros of $(s-1)^2(-s+2)^2 + (s+1)^2(s+2)^2$

$$= (s^2-2s+1)(s^2-4s+4) + (s^2+2s+2)(s^2+4s+4)$$

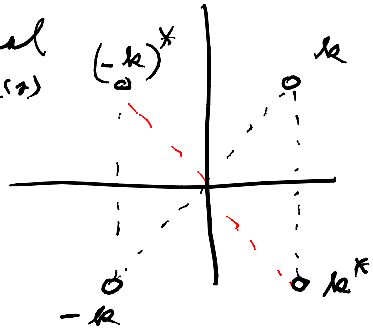
$$= 2[s^4 + 4s^2 + 8s^2 + s^2 + 4] = 2[s^4 + 13s^2 + 4]$$

$x = s^2 \Rightarrow x^2 + 13x + 4 = 0$; $x = -\frac{13}{2} \pm \sqrt{\left(\frac{13}{2}\right)^2 - 4}$

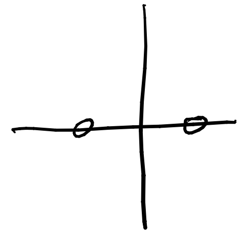
$$s_{1,2,3,4} = \pm\sqrt{-\frac{13}{2} \pm \sqrt{\left(\frac{13}{2}\right)^2 - 4}}$$



In general zeroes of $\mathrm{Ev}\, y(s)$



or



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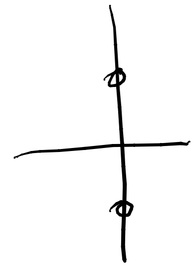


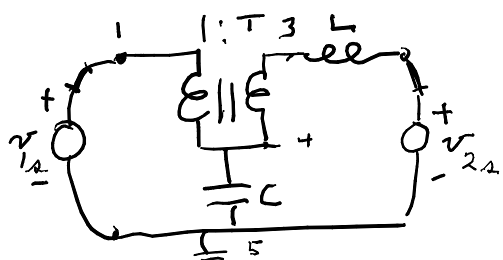
In Richards' function notes, let $k_2 = k_1^*$

coeff. of s^2 in numerator $\dfrac{k_1 y(k_2) - k_2 y(k_1)}{k_1 k_2 [k_1 y(k_1) - k_2 y(k_2)]}$

$$= \frac{k_1 y(k_1^*) - k_1^* y(k_1)}{|k_1|^2 [k_1 y(k_1) - k_1^* y(k_1^*)]}$$

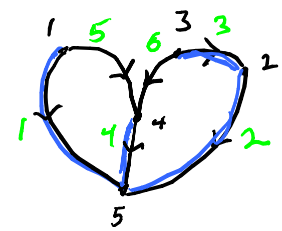
$y(s^*) = y^*(s)$ if PR as real coefficients (if rational)

$$= \frac{k_1 y^*(k_1) - (k_1 \cancel{y}(k_1))^*}{|k_1|^2 [k_1 y(k_1) - (k_1 y(k_1))^*]} = \frac{2\cancel{j}\, \mathrm{Im}(k_1 y^*(k_1))}{|k_1|^2 \times 2\cancel{j}\, \mathrm{Im}(k_1 y(k_1))} = \text{purely real}$$



choose output

$$y = \begin{bmatrix} -i_1 \\ -i_2 \end{bmatrix}$$



here $i_1 = -i_5 = -x_5$, $i_2 = -i_6 = -x_6$

$=$ semistate $= x = \begin{bmatrix} v_t \\ \hline i_\ell \end{bmatrix}$

$y = \text{output} = \begin{bmatrix} -x_5 \\ -x_6 \end{bmatrix}$; $\quad y = Cx = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$

changed x state to x_{st}

In terms of the state:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1-T & 0 \\ 0 & -T \end{bmatrix} \begin{bmatrix} x_4 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ T & -1 \\ 0 & 0 \end{bmatrix} u \Rightarrow x_6 = x_{2st}$$

$$x_5 = -T x_6 = -T x_{2st}$$

$$-y = \begin{bmatrix} 0 & +T \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_{1\,state} = x_4 \\ x_{2\,state} = x_6 \end{bmatrix} = \begin{bmatrix} -x_5 \\ -x_6 \end{bmatrix}$$

or

$$\underset{\text{output}}{y} = -\begin{bmatrix} 0 & +T \\ 0 & -1 \end{bmatrix} x_{st} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u = Cx_{st} + D$$

$$\frac{d}{dt}\begin{bmatrix} x_4 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{T+1}{C} \\ \frac{T-1}{L} & 0 \end{bmatrix} \begin{bmatrix} x_4 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\frac{T}{L} & 1/L \end{bmatrix} u \quad \Big\} \text{ state eqs. from last time}$$

$$y = -\begin{bmatrix} 0 & +T \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_4 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u \quad \Big\} \text{ from line above}$$

To get the transfer function take Laplace transform

$$\underset{\text{2 port}}{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}} = \mathcal{L}[y] = -\begin{bmatrix} 0 & +T \\ 0 & -1 \end{bmatrix} \left[s\mathbb{1}_2 - \begin{bmatrix} 0 & -\frac{T+1}{C} \\ \frac{T-1}{L} & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 0 & 0 \\ -\frac{T}{L} & \frac{1}{L} \end{bmatrix} \mathcal{L}[u], \quad \mathcal{L}[u] = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore \underset{\substack{\text{2 port} \\ \text{admittance}}}{Y(s)} = -\begin{bmatrix} 0 & T \\ 0 & -1 \end{bmatrix} \begin{bmatrix} s & \frac{T-1}{C} \\ \frac{1-T}{L} & s \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \\ -\frac{T}{L} & \frac{1}{L} \end{bmatrix} \; ; \; \det(s\mathbb{1}_2 - A) = s^2 + \frac{(T-1)^2}{LC}$$

$$= -\begin{bmatrix} 0 & T \\ 0 & -1 \end{bmatrix} \frac{1}{s^2 + \frac{(T-1)^2}{LC}} \begin{bmatrix} s & \frac{1-T}{C} \\ \frac{T-1}{L} & s \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -\frac{T}{L} & \frac{1}{L} \end{bmatrix}$$

$$= \frac{-1}{s^2 + \frac{(T-1)^2}{LC}} \begin{bmatrix} \frac{T^2-T}{L} & sT \\ \frac{1-T}{L} & -s \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -\frac{T}{L} & \frac{1}{L} \end{bmatrix} = \frac{-1}{s^2 + \frac{(T-1)^2}{LC}} \begin{bmatrix} -sT^2/L & sT/L \\ \frac{sT}{L} & -\frac{s}{L} \end{bmatrix}$$

$$= \frac{s/L}{s^2 + \frac{(T-1)^2}{LC}} \begin{bmatrix} T^2 & -T \\ -T & 1 \end{bmatrix} \times (+1) \qquad = \frac{s/L}{s^2 + \frac{(T-1)^2}{LC}} \begin{bmatrix} -T \\ 1 \end{bmatrix} \begin{bmatrix} -T & 1 \end{bmatrix}$$