Homework now due on W

EE 610
10/01/07

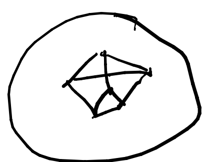
minimal here means least number of components

passivity $\equiv$ positive real function

(can not get more energy out than put in)

$z(s) = -2s$ at $s = 1$, $\mathrm{Re}\, z(s) = -2 < 0$ so not positive-real

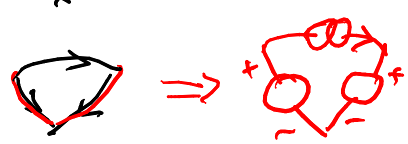
For a finite graph; $v_b^T \cdot i_b = 0$ as = power into the "graph" from the outside



$$\left.\begin{aligned} v_b &= \mathcal{C}^T v_t \\ i_b &= \mathcal{J}^T i_\ell \end{aligned}\right\} \text{ into } v_b^T i_b = (v_t^T \cdot \mathcal{C})(\mathcal{J}^T i_\ell) = 0 \text{ from}$$

but can find a graph with v_t freely chosen & same for i_ℓ

here then can cancel v_t & i_ℓ



$\Rightarrow \quad \mathcal{C}\mathcal{J}^T = 0_{t \times \ell}$

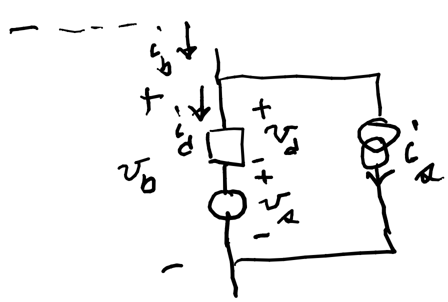
Now consider 2 circuits with the same graph; N & $\hat{N}$

$$\underbrace{v_t^T \mathcal{C}}_{= v_b^T} \underbrace{\mathcal{J}^T \hat{i}_\ell}_{= \hat{i}_b} = 0$$

v_b, i_b $\qquad$ $\hat{v}_b, \hat{i}_b$

$v_b = \mathcal{C} v_t$ $\qquad$ $\hat{i}_b = \mathcal{J}^T \hat{i}_\ell$

$\Rightarrow \quad v_b^T \cdot \hat{i}_b = 0 \quad \Rightarrow$ Tellegen's Theorem



$i_b = i_s + i_d$

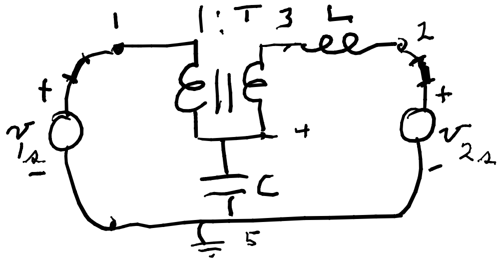
$v_b = v_s + v_d$

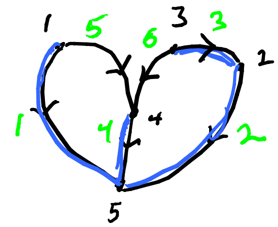
d = device

s = independent source

$A v_d = B i_d$

$v_b = \mathcal{C}^T v_t \,, \quad i_b = \mathcal{J}^T i_\ell$





— = tree

here $b=6,\ t=4,\ \ell=2$

laws of transformer

$i_{5_d}+Ti_{6_d}=0$

(Σ of Amp turns $=0$)

$v_{6_d}=Tv_{5_d}$

$$\begin{array}{l} \text{shorts}\ \{ \\ \\ \text{inductor} \rightarrow \\ \text{capacitor} \rightarrow \\ \Sigma AT=0 \rightarrow \\ \text{Volt step up} \rightarrow \end{array} \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&sC&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&-T&1 \end{bmatrix} \begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6 \end{bmatrix}_d = \begin{bmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&sL&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&T\\ 0&0&0&0&0&0 \end{bmatrix} \begin{bmatrix} i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6 \end{bmatrix}_d$$

$$v_s=\begin{bmatrix} v_{1s}\\ v_{2s}\\ 0\\ 0\\ 0\\ 0 \end{bmatrix};\quad i_s=\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

Now

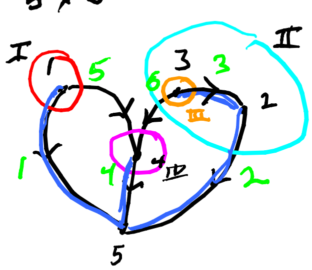
$$v_d = v_b - v_s = e^T v_t - v_s$$
$$i_d = i_b - i_s = \mathcal{J}^T i_\ell - i_s$$

$$A v_d = B i_d$$
$$\Downarrow$$
$$A[e^T v_t - v_s] = B[\mathcal{J}^T i_\ell - i_s]$$

$$A e^T v_t - B\mathcal{J}^T i_\ell = A v_s - B i_s$$

$$= \underbrace{\left[A e^T \ \vdots\ -B\mathcal{J}^T\right]}_{b\times b} \underbrace{\begin{bmatrix} v_t\\ i_\ell \end{bmatrix}}_{b\text{-vector}} = A v_s - B i_s$$

let $x=\begin{bmatrix} v_t\\ i_\ell \end{bmatrix}$ (semistate vector)



cut set eqs.

$$\begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} = \left[\begin{array}{cccc|cc} 1&0&0&0&1&0\\ 0&1&0&0&0&1\\ 0&0&1&0&0&1\\ 0&0&0&1&-1&-1 \end{array}\right] \begin{bmatrix} i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6 \end{bmatrix}_b$$

tie set eqs.

$$\begin{bmatrix} 0\\ 0 \end{bmatrix} = \left[\begin{array}{cccc|cc} -1&0&0&1&1&0\\ 0&-1&-1&1&0&1 \end{array}\right] \begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6 \end{bmatrix}_b$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -T & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix} = A\mathcal{C}^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & sC \\ 0 & 0 & 0 & 0 \\ -T & 1 & 1 & T-1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sL & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & +T \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & -1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = B\mathcal{J}^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -sL \\ 1 & 1 \\ 1 & T \\ 0 & 0 \end{bmatrix}$$

$$\left[A\mathcal{C}^T \,\middle|\, -B\mathcal{J}^T\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & sL \\ 0 & 0 & 0 & sC & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -T \\ -T & 1 & 1 & T-1 & 0 & 0 \end{bmatrix}$$

multiplies $x = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_5 \\ i_6 \end{bmatrix}_b = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$

$$A v_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{1s} \\ v_{2s} \end{bmatrix}$$

The circuit equations are $[A\mathcal{C}^T \; -B\mathcal{J}^T]x = A v_s$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & sL \\ 0 & 0 & 0 & sC & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -T \\ -T & 1 & 1 & T-1 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{1s} \\ v_{2s} \end{bmatrix}$$

gives 6 eqs. & 6 unknown

Rewrite to put s multiplied terms on the left, $s = \frac{d}{dt}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L \\ 0 & 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & T \\ T & -1 & -1 & 1-T & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u,$$

u = input vector $= \begin{bmatrix} v_{1s} \\ v_{2s} \end{bmatrix}$

in the form
$$\boxed{\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}}$$

let y = output
assume linear combinations of branch voltages & currents

generic form for circuit differential equations
call these the semi-state equations