$$P_{ave}(j\omega) = Re(I^{*T}V)\Big|_{s=j\omega} \qquad * = j \to -j$$

$$= \frac{(I^{*T}V + (I^{*T}V)^*)}{2}\Big|_{s=j\omega} = \frac{I^{*T}V + (I^{T}V^*)}{2}\Big|_{s=j\omega}$$

but $I = YV$

$$= \frac{(YV)^{*T}V + ((YV)^{T}V^*)^{T}}{2}\Big|_{s=j\omega}$$

$$= \frac{V^{*T}Y^{*T}V + V^{*T}(Y)V}{2}\Big|_{s=j\omega} = V^{*T}\left[\frac{Y^{*T}+Y}{2}\right]V\Big|_{s=j\omega}$$

$= 0$ if lossless for all complex vectors V

$$Y(j\omega)^{*T} + Y(j\omega) = 0_{matrix}$$

if $Y(s)$ is rational with real coefficients

$$= \left[Y(-s)^T + Y(s)\right]\Big|_{s=j\omega} = 0_{matrix}$$

then $Y(j\omega)^* = Y(-j\omega) = Y(-s)\Big|_{s=j\omega}$

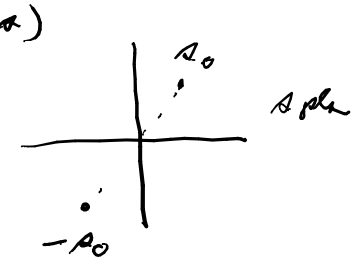
if stable $Y(s)$ is analytic in $Re\, s > 0$ & here $Y(-s)^T + Y(s)$ is zero on an infinite set & if $Y(s)$ is rational, by complex variable theory this sum is zero for all s.

$$Y(-s)^T + Y(s) \equiv 0 \quad \text{for all } s \Leftarrow \text{lossless condition}$$

If $Y(s)$ is $1\times 1 \Rightarrow$ driving point $Y(s) = y(s)$

$$2\,Ev(y(s)) = y(s) + y(-s) \equiv 0$$

"
Even



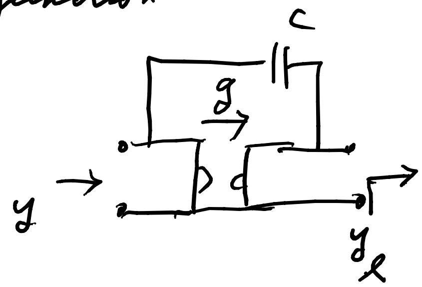
Ex: $\quad y(s) = \dfrac{2s}{s^2+7}$;

$s \to -s =$ Hurwitz conjugate

$$2\,Ev\, y(s) = y(s) + y(-s) = y + y_*$$

$$= \frac{2s}{s^2+7} + \frac{(-2s)}{(-s)^2+7} = 0$$

We can synthesize a lossless $y(s)$ by the Richards' function



$$\frac{y_\ell}{g} = \frac{1-\left(\frac{Cs}{g}\right)\left(\frac{y_{in}(s)}{g}\right)}{\left(\frac{y_{in}(s)}{g}\right)-\left(\frac{Cs}{g}\right)}$$ from 09/10/07

P.361 $$R(s) = \frac{k\,Z(s) - s\,Z(k)}{k\,Z(k) - s\,Z(s)} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}$$

$$= \frac{k\frac{1}{y(s)} - s\frac{1}{y(k)}}{k\frac{1}{y(k)} - \frac{s}{y(s)}} = \frac{k\,y(k)\left(1 - \frac{s}{k}\cdot\frac{y(s)}{y(k)}\right)}{k\,y(k)\left(\frac{y(s)}{y(k)} - \frac{s}{k}\right)} = \frac{1-\left(\frac{s}{k}\right)\left(\frac{y(s)}{y(k)}\right)}{\left(\frac{y(s)}{y(k)}\right)-\left(\frac{s}{k}\right)}$$

choose $g = y(k)$, $\frac{s}{k} = \frac{Cs}{g} \Rightarrow \frac{1}{k} = \frac{C}{y(k)} \Rightarrow C = \frac{y(k)}{k}$

$s=k$ gives a pole & zero cancellation & can get another if $s=-k$ also can be used to cancel choose k to be a zero of $Ev\,y(s)$, then $y(-s) = -y(s)$

$$k\,y(k) - s\,y(s)\Big|_{s=-k} = k\,y(k) - (-k)(+y(-k)) \text{ if } s=-k \text{ is a zero of } Ev\,y(s)$$
$$= k[y(k)+y(-k)] = 0$$

two $$k\,y(s) - s\,y(k)\Big|_{s=-k} = k\,y(-k) - (-k)\,y(k) = k[y(-k)+y(k)] = 0 \text{ if } k = \text{a zero of } Ev\,y(s)$$

But if $y(s)$ is lossless any real positive k is a zero of $Ev\,y(s)$.

Ex: $y(s) = \frac{2s}{s^2+7}$ choose any pos. real k, say $k=2$

$$y(k) = y(2) = \frac{4}{4+7} = 4/11$$

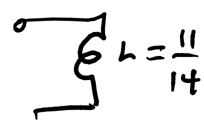
$g = y(k) = 4/11$ $C = \frac{y(k)}{k} = \frac{4/11}{2} = 2/11$

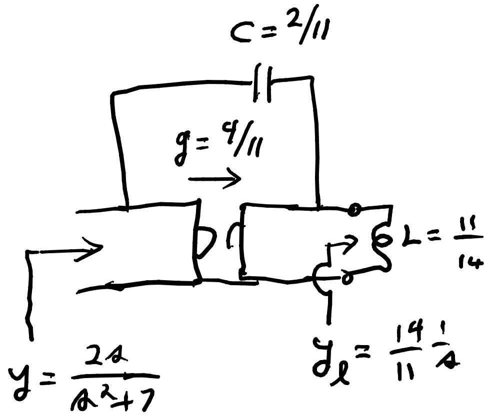
$$\frac{y_\ell(s)}{g} = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} = \frac{2\times\frac{4}{11} - s\,\frac{2s}{s^2+7}}{2\times\frac{2s}{s^2+7} - s\times\frac{4}{11}} = \frac{\frac{8}{11}s^2 + \frac{7\times 8}{11} - 2s^2}{4s - \frac{4s^3}{11} - \frac{28s}{11}}$$

we know $(s-2)(s-(-2)) = s^2-4$ cancels in numerator and denominator.

$$\frac{y_\ell(s)}{4/11} = \frac{(8-22)s^2 + 7\times 8}{-4s^3 + (44-28)s} = \frac{1}{s}\times\frac{-14\left[s^2 - \frac{56}{14}\right]}{-4\left[s^2 - \frac{16}{4}\right]} = \frac{1}{s}\cdot\left(\frac{7}{2}\right)$$

$$y_\ell(s) = \frac{4}{11}\times\frac{7}{2}\cdot\frac{1}{s} = \frac{14}{11}\cdot\frac{1}{s}$$

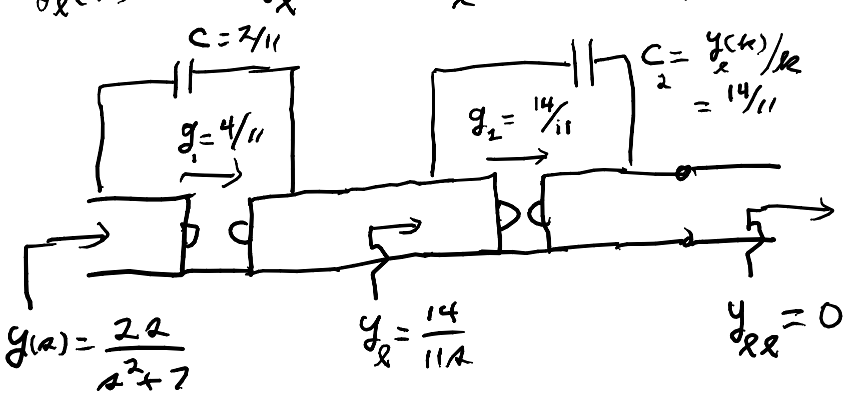




$y_\ell(s) = \frac{14}{11s}$, by Richards' function, choose a k, say $k=1$

$$\frac{y_{\ell\ell}(s)}{y_\ell(k)} = \frac{k\,y_\ell(k) - s\,y_\ell(s)}{k\,y_\ell(s) - s\,y_\ell(k)} = \frac{1\cdot\frac{14}{11} - \frac{14}{11}}{1\cdot\frac{14}{11s} - s\cdot\frac{14}{11}} \equiv 0 \Rightarrow \text{open circuit}$$

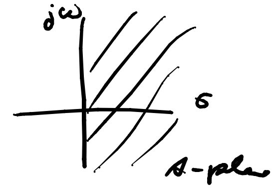
as $i = yv = 0$



Positive-real function $f(s)$

(for $y(s)$ & $z(s)$)

1) analytic in $\mathrm{Re}\, s > 0$
(stable circuit)



$s = \sigma + j\omega$

2) $f(\sigma)$ is real in $\mathrm{Re}\, s > 0$
(real components)

3) $\mathrm{Re}\, f(s) \geq 0$ in $\mathrm{Re}\, s > 0$
(passive)

importance: any passive (real) circuit has $y(s)$ & $z(s)$ positive real