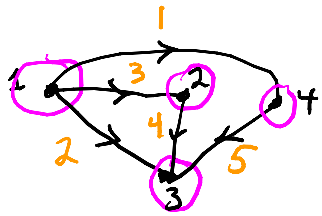
P.124 3.12 $n_T = \det(A_{\cancel{a}} A^T_{\cancel{a}})$

EE 610
09/19/07

augmented incidence matrix A_a



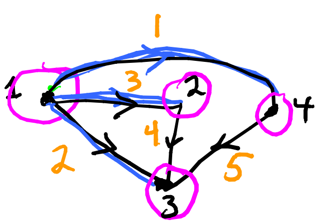
$$A_a = \underbrace{\left.\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}\right\} n}_{b}$$

remove row 3 $\Rightarrow$ node 3 = reference node

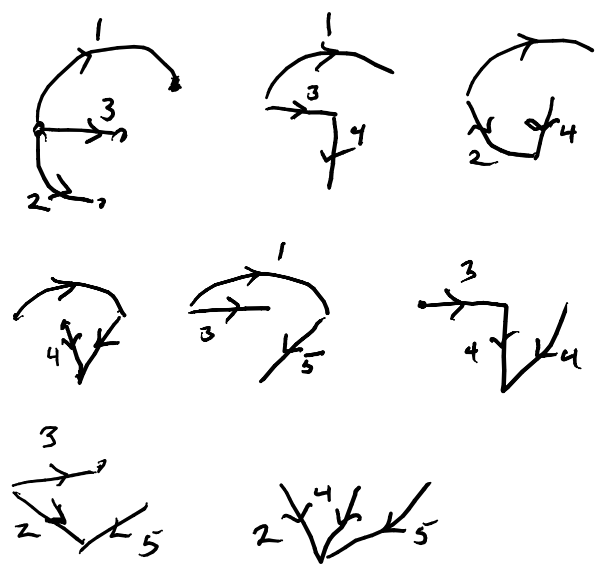
$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

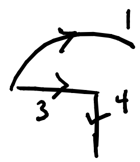
$$n_T = \det(AA^T) = 12 - 2 - 2 = 8$$



trees $\Rightarrow$

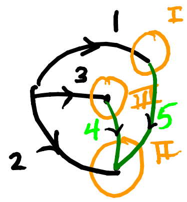


choose a tree, say branches 1, 3, 4



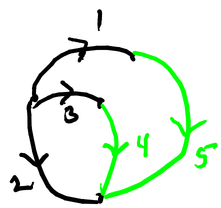
$\Rightarrow$ go to a new numbering with branches 1, 2, 3 in the tree

use



KCL $\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}1&0&0&0&-1\\0&1&0&1&1\\0&0&1&-1&0\end{bmatrix}\begin{bmatrix}i_1\\i_2\\i_3\\i_4\\i_5\end{bmatrix}_b \Rightarrow \underline{0}_t = \mathcal{C}\, i_b$ KCL

$\mathcal{C} = \left[\begin{array}{c|cc} & 0 & -1\\ 1_3 & 1 & 1\\ & -1 & 0\end{array}\right]$, $\mathcal{C}$ = cut-set matrix



KVL $\begin{bmatrix}0\\0\end{bmatrix} \begin{array}{l} = v_4 - v_2 + v_3 = 0\\ = v_5 - v_2 + v_1 = 0\end{array} = \left[\begin{array}{ccc|cc}0&-1&1&1&0\\1&-1&0&0&1\end{array}\right]\begin{bmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{bmatrix}_b$

$\underline{0}_\ell = \mathcal{T}\cdot \underline{v}_b$ $\quad \mathcal{T}$ = tie-set matrix

$\mathcal{T} = \left[\begin{array}{ccc|c}0&-1&1& \\ 1&-1&0& 1_2\end{array}\right]$

$\mathcal{C} = \left[\, 1_t \;\middle|\; K_\ell \,\right]$

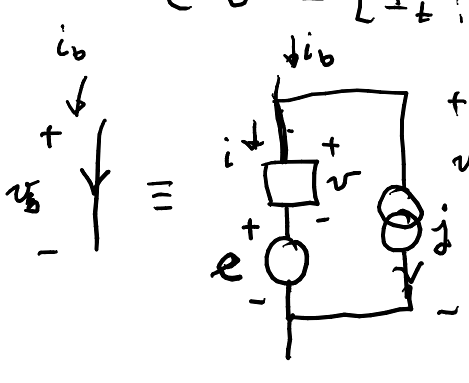
$\mathcal{T} = \left[\, K_t \;\middle|\; 1_\ell \,\right]$

$\mathcal{C} = \underbrace{\left[\, 1_t \;\middle|\; \overbrace{K_\ell}^{\ell} \,\right]}_{b} \Big\}\, t \qquad \& \quad K_\ell = t \times \ell = \begin{bmatrix}0&-1\\1&1\\-1&0\end{bmatrix}$

$\mathcal{T} = \underbrace{\left[\, K_t \;\middle|\; 1_\ell \,\right]}_{\underbrace{}_{t}\; b \;\underbrace{}_{\ell}} \Big\}\, \ell, \quad K_t = \begin{bmatrix}0&-1&1\\1&-1&0\end{bmatrix} = \ell \times t$

note $K_t = -K_\ell^T$ $\Rightarrow$ KCL $\Rightarrow$ KVL & KVL $\Rightarrow$ KCL

$\mathcal{C}\,\mathcal{T}^T = \left[\, 1_t \;\middle|\; K_\ell \,\right]\begin{bmatrix}K_t^T = -K_\ell\\ 1_\ell\end{bmatrix} = -K_\ell + K_\ell = 0_{t\times\ell}$



$i_b = i + j$

$v_b = v + e$

assume an admittance law: $i = Y_b v$

but $v_b = e^T v_t$, $i_b = \sigma^T i_\ell$

$$0_\ell = \sigma v_b = [K_t \mid 1_\ell]\begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = 0_\ell = K_t v_t + v_\ell \Rightarrow v_\ell = -K_t v_t = K_\ell^T v_t$$

$$v_t = 1_t v_t$$

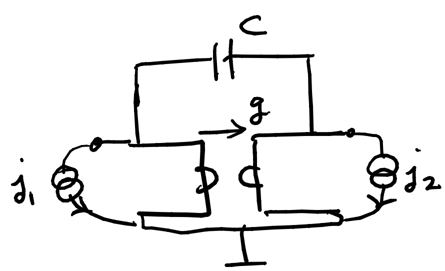
$$v_b = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = \begin{bmatrix} 1_t \\ K_\ell^T \end{bmatrix} v_t = e^T v_t = v_b$$

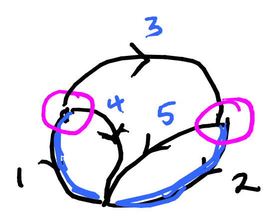
$$i_b = \sigma^T i_\ell = i + j = Y_b v_b + j = Y_b e^T v_t + j$$

but $e\, i_b = 0_t$; $\Rightarrow e\, i_b = 0 = e\sigma^T i_\ell = e Y_b e^T v_t + e j$

$$-ej = e Y_b e^T v_t$$ gives t eqs. in t unknowns v_t given current sources j

Ex:





$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & +1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}_b \qquad Y_b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & -g \\ 0 & 0 & 0 & +g & 0 \end{bmatrix}$$

$$= e \cdot i_b$$

$$e Y_b e^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & 0 & -g \\ 0 & 0 & 0 & g & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ sC & -sC \\ 0 & -g \\ g & 0 \end{bmatrix}$$

$$= \begin{bmatrix} sC & -sC-g \\ -sC+g & sC \end{bmatrix} , \quad -ej = \begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j_1 \\ -j_2 \end{bmatrix}$$

$$\begin{bmatrix} -i_1 \\ -i_2 \end{bmatrix} = \begin{bmatrix} sC & -sC-g \\ -sC+g & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \text{2-port admittance description}$$