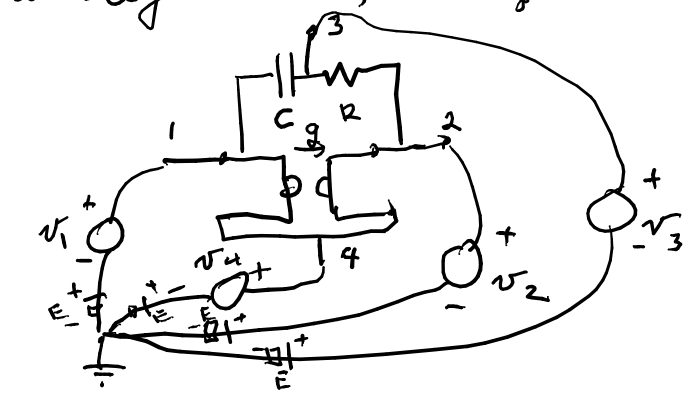
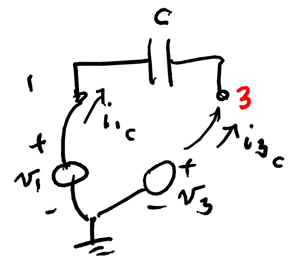
Indefinite Y, Y_{indef}

EE 610
09/17/07



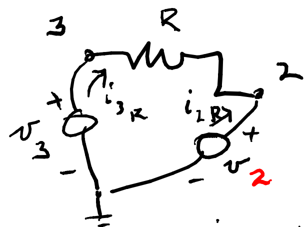
$$i = Y_{indef}\left(v + E\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\right)$$

sum of entries in a row is zero & same for sum of entries in a column of Y_{indef}.



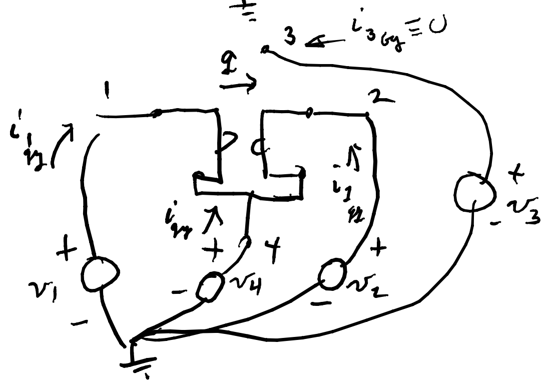
$$i_{1_c} = sC(v_1 - v_3)$$
$$i_{3_c} = sC(v_3 - v_1)$$

$$Y_{indef_C} = \begin{bmatrix} sC & 0 & -sC & 0 \\ 0 & 0 & 0 & 0 \\ -sC & 0 & sC & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



$$i_{2_R} = G(v_2 - v_3) \qquad G = 1/R$$
$$i_{3_R} = G(v_3 - v_2)$$

$$Y_{ind_R} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G & -G & 0 \\ 0 & -G & G & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



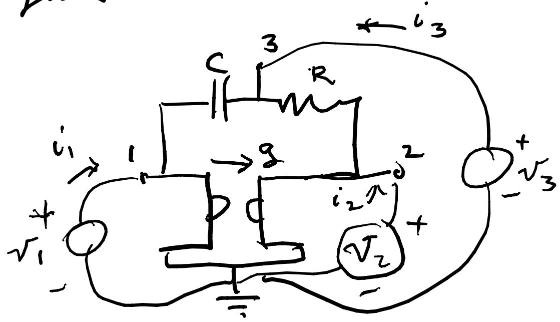
$$i_{1_{gy}} = -g(v_2 - v_4)$$
$$i_{2_{gy}} = g(v_1 - v_4)$$
$$i_{3_{gy}} = 0$$
$$i_{4_{gy}} = -i_{1_{gy}} - i_{2_{gy}} = -[-g(v_2 - v_4)] - g[v_1 - v_4]$$
$$= -g v_1 + g v_2$$

$$Y_{ind_{gy}} = \begin{bmatrix} 0 & -g & 0 & g \\ g & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \\ -g & g & 0 & 0 \end{bmatrix}$$

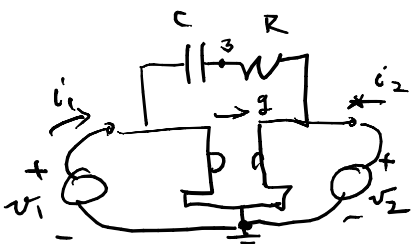
$$Y_{indef}^{(A)} = \begin{bmatrix} sC & -g & -sC & g \\ g & G & -G & -g \\ -sC & -G & sC+G & 0 \\ -g & g & 0 & 0 \end{bmatrix} \qquad i = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = Y_{ind} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

move ground to node 4 $\Rightarrow v_4 = 0$ can scratch out last column as multiplies 0 ; ignore i_4 as $i_4 = -i_1 - i_2 - i_3$

gives the nodal admittance, $Y_{node} = \begin{bmatrix} sC & -g & -sC \\ g & G & -G \\ -sC & -G & sC+G \end{bmatrix}$



$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = Y_{node} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



here $i_3 = 0$ but $v_3 \neq 0$; eliminate v_3 from Y_{node} gives $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$i_3 = 0 = -sC\, v_1 - G\, v_2 + (sC+G) v_3 \Rightarrow v_3 = \frac{sC}{sC+G} v_1 + \frac{G}{sC+G} v_2$$

eliminate v_3 from 1st 2 rows:

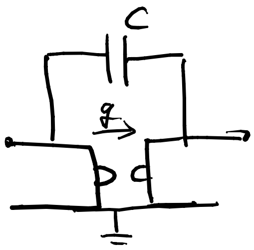
$$i_1 = sC v_1 - g v_2 - sC \left[\frac{sC}{sC+G} v_1 + \frac{G}{sC+G} v_2 \right] =$$

$$= \frac{sCG}{sC+G} v_1 - \left[g + \frac{sCG}{sC+G} \right] v_2$$

$$i_2 = g v_1 + G v_2 - G \left[\frac{sC}{sC+G} v_1 + \frac{G}{sC+G} v_2 \right]$$

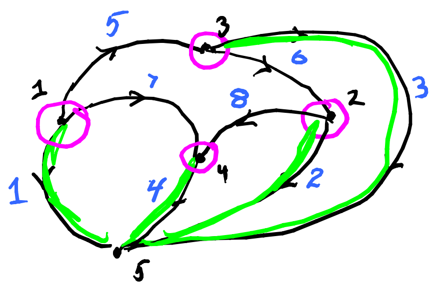
$$= \left[g - \frac{sCG}{sC+G} \right] v_1 + \frac{sCG}{sC+G} v_2$$

$$Y = \begin{bmatrix} \frac{sCG}{sC+G} & -g - \frac{sCG}{sC+G} \\ g - \frac{sCG}{sC+G} & \frac{sCG}{sC+G} \end{bmatrix} \qquad \text{if } G \to \infty \quad Y_{(A)} = \begin{bmatrix} sC & -g - sC \\ g - sC & sC \end{bmatrix}$$



is the above with $R = 0$

graph for original circuit



b = # of branches = 8

n = # of nodes = 5

t = # of tree branches = $n-1$
(for a chosen tree)

= green branches

= $4 = 5-1$

one "separate part"

l = cotree branches = $n - s$ if s separate parts

= links = $b - t$

= $b - (n-1)$

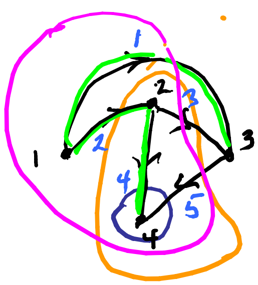
1 sep. part

t = # of independent current equations = KCL's

l = # of independent voltage law eqs = KVL's

KCL = $0 = C\, i_b$; i_b = branch currents, a b-vector

C = cut set matrix, $t \times b$ matrix



$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$ = 3×5 matrix

gives 3 linearly independent eqs

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = C \begin{bmatrix} i_{b_1} \\ i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \end{bmatrix}$$