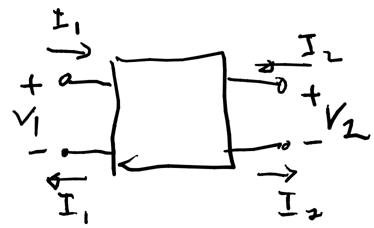
EE 610
09/10/07

admittance = Y

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



y = chain

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \Rightarrow y_{21}V_1 = I_2 - y_{22}V_2 \Rightarrow V_1 = \frac{1}{y_{21}}\left(-(-I_2) - y_{22}V_2\right)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 = y_{11}\left\{\frac{1}{y_{21}}\left[-(-I_2) - y_{22}V_2\right]\right\} + y_{12}V_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\Delta_y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Leftarrow$$ y as a function of Y

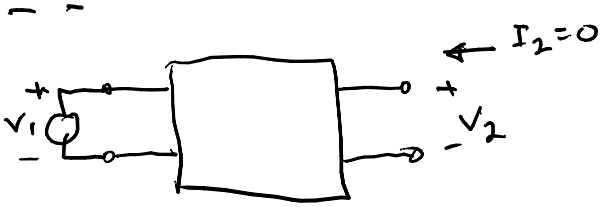
$$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$$

$$V_1 = AV_2 - BI_2 \Rightarrow +BI_2 = AV_2 - V_1 \Rightarrow I_2 = \frac{1}{B}(-V_1 + AV_2)$$

$$I_1 = CV_2 - DI_2 = CV_2 - D\left\{\frac{-1}{B}V_1 + \frac{A}{B}V_2\right\} = +\frac{D}{B}V_1 + \left(\frac{BC - AD}{B}\right)V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & -\frac{\Delta_N}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Leftarrow$$ Y as a function of y

$$\Delta_N = AD - BC$$



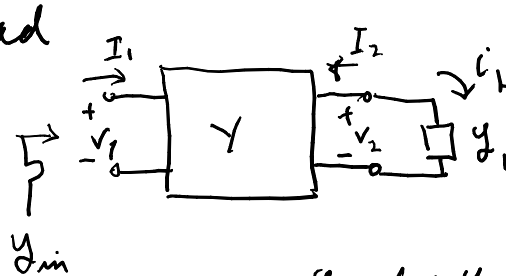
$$V_1 = AV_2 + B(-I_2)\Big|_{I_2=0} = AV_2$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{1}{A}$$ when $I_2 = 0$ (an open circuit load)

in terms of Y

$$I_2 = 0 = y_{21}V_1 + y_{22}V_2 \Rightarrow \frac{V_2}{V_1}\Big|_{I_2=0} = -\frac{y_{21}}{y_{22}}$$

Now load



$i_L = -I_2$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad -y_L V_2 = I_2$$

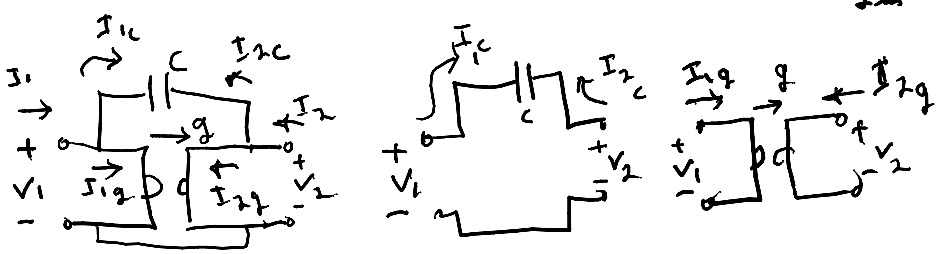
$$-y_L V_2 = y_{21} V_1 + y_{22} V_2 \Rightarrow (-y_L - y_{22}) V_2 = y_{21} V_1$$

$$\frac{V_2}{V_1} = -\frac{y_{21}}{y_{22}+y_L} \Rightarrow I_1 = y_{11} V_1 - \frac{y_{12}\, y_{21}}{y_{22}+y_L} \cdot V_1 = \left(\frac{y_{11} y_{22} + y_{11} y_L - y_{12} y_{21}}{y_{22}+y_L} \right) V_1$$

$$\frac{I_1}{V_1} = y_{in} = \frac{\Delta y + y_{11} y_L}{y_{22} + y_L}$$

next find y_L in terms of y_{in}: $y_{in}(y_{22}+y_L) = \Delta y + y_{11} y_L$

$$(y_{in} - y_{11}) y_L = \Delta y - y_{22} y_{in} \Rightarrow y_L = \frac{\Delta y - y_{22} y_{in}}{y_{in} - y_{11}}$$



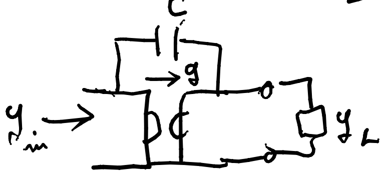
$I_1 = I_{1c} + I_{1g}$

$I_2 = I_{2c} + I_{1g}$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_c = \begin{bmatrix} cs & -cs \\ -cs & cs \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_g = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_c + \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_g = \left\{ \begin{bmatrix} cs & -cs \\ -cs & cs \end{bmatrix} + \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \right\} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} cs & -cs-g \\ -cs+g & cs \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

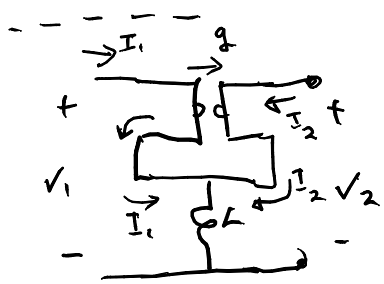


$$, \Delta y = (cs)^2 - [-(cs+g)(-cs+g)]$$
$$(cs)^2 - [(cs)^2 - g^2] = g^2$$

$$y_{in} = \frac{\Delta_y + y_{11} y_L}{y_{22} + y_L} = \frac{g^2 + Cs \cdot y_L}{Cs + y_L}$$

$$y_L = \frac{\Delta_y - y_{22} y_{in}}{y_{in} - y_{11}} = \frac{g^2 - Cs \cdot y_{in}}{y_{in} - Cs} = \frac{g^2}{g^2}\left[\frac{1 - \left(\frac{C}{g}s\right)\cdot \frac{y_{in}}{g}}{\frac{1}{g}\left(\frac{y_{in}}{g} - \frac{Cs}{g}\right)}\right.$$

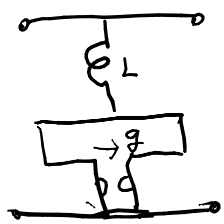
$$\frac{y_L}{g} = \frac{1 - \left(\frac{Cs}{g}\right)\left(y_{in}/g\right)}{\left(y_{in}/g\right) - \left(Cs/g\right)}$$ $\Rightarrow$ leads to a Richards' function (P. I. Richards)



here the Z matrices add $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$Z = \frac{1}{g^2}\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + \begin{bmatrix} sL & sL \\ sL & sL \end{bmatrix}$$

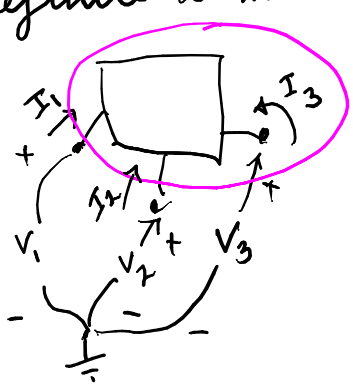
$$= \begin{bmatrix} sL & \frac{1}{g} + sL \\ -\frac{1}{g} + sL & sL \end{bmatrix}; \quad \Delta_Z = \frac{1}{g^2}$$



here Z_{gyr} no longer holds as $\frac{V_1}{g} = \frac{V_2}{g}$ for the bottom which destroys the law giving Z_{gyr}

need the same law for the sub-circuits before and after connect.

Indefinite admittance; sum of entries in any row add to zero & the same for any column.



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = Y_{ind}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

by KCL: $I_1 + I_2 + I_3 = 0$