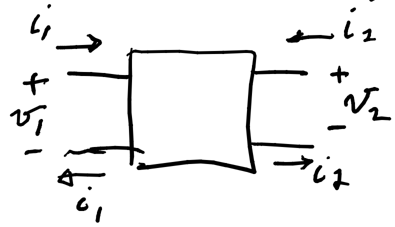
EE610
09/05/07

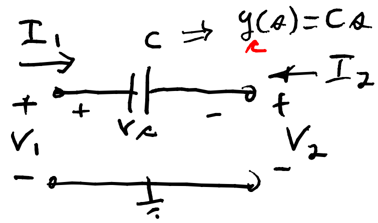
admittance matrix 2-port



$P_{in}(t) = v_1(t)\, i_1(t) + v_2(t)\, i_2(t)$

$s = \sigma + j\omega$, d/dt, Laplace transform variable, s in e^{st}

$Y(s)\, V(s) = I(s)$; $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

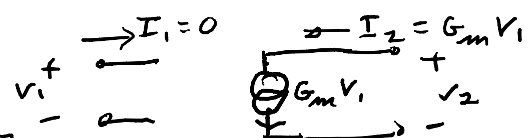


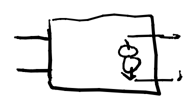
$I_1 = -I_2$

$V_1 = V_c + V_2 = \dfrac{I_c}{y_c(s)} + V_2 = \dfrac{I_1}{Cs} + V_2$

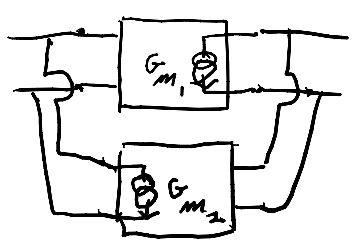
$I_1 = Cs(V_1 - V_2)$
$I_2 = -Cs(V_1 - V_2)$ $\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Cs & -Cs \\ -Cs & +Cs \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow Y(s) = Cs \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

VCCS ⇒ G of Spice





$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ G_m & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

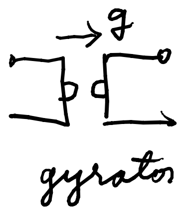


$Y = Y_{G_{m_1}} + Y_{G_{m_2}} = \begin{bmatrix} 0 & 0 \\ G_{m_1} & 0 \end{bmatrix} + \begin{bmatrix} 0 & G_{m_2} \\ 0 & 0 \end{bmatrix}$

useful to choose $G_{m_2} = -G_{m_1} = g$

since $P_{in}(t) = v_1 i_1 + v_2 i_2 = v_1 \cdot G_{m_2} v_2 + v_2 \cdot G_{m_1} v_1 = v_2 v_1 (G_{m_2} + G_{m_1})$

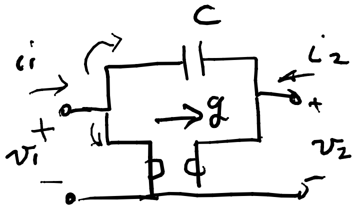
$= 0$ if $G_{m_2} = -G_{m_1}$



$$\Rightarrow \quad Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = -Y^T$$

(skew-symmetric matrix)

$\Rightarrow$ basic nonreciprocal element



$$Y = Y_{cap} + Y_{gyr} = \begin{bmatrix} Cs & -Cs \\ -Cs & Cs \end{bmatrix} + \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$$

$$= \begin{bmatrix} Cs & -g-Cs \\ g-Cs & Cs \end{bmatrix} \; ; \; \det Y = (Cs)^2 - [-(g+Cs)(g-Cs)]$$

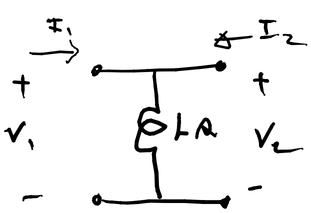
$$= (Cs)^2 + g^2 - (Cs)^2$$

$$= g^2$$

$$V = ZI, \quad Z = Y^{-1} = \frac{1}{g^2}\begin{bmatrix} Cs & (-1)^3(-g-Cs) \\ (-1)^3(g-Cs) & Cs \end{bmatrix}$$

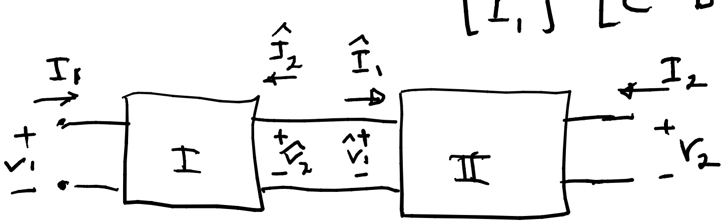
$$= \begin{bmatrix} s(C/g^2) & s(C/g^2) + 1/g \\ s(C/g^2) - 1/g & s(C/g^2) \end{bmatrix} = s\left(\frac{C}{g^2}\right)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1/g \\ -1/g & 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow Z_L(s) = \begin{bmatrix} s(C/g^2) & s(C/g^2) \\ s(C/g^2) & s(C/g^2) \end{bmatrix}$$



if $L = C/g^2$

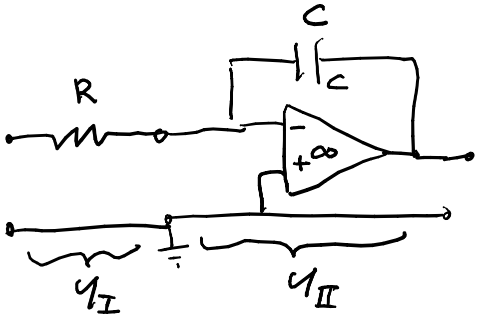
chain matrix: $\mathcal{A} \Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

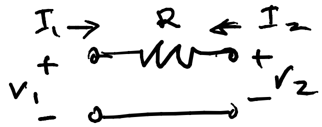


$$\hat{V}_1 = \hat{V}_2$$
$$\hat{I}_1 = -\hat{I}_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{A}_{I}\begin{bmatrix} \hat{V}_2 \\ -\hat{I}_2 \end{bmatrix} = \begin{bmatrix} \hat{V}_1 \\ \hat{I}_1 \end{bmatrix} = \mathcal{A}_{II}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{A}_{I}\cdot\mathcal{A}_{II}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\mathcal{A} = \mathcal{A}_{I}\cdot\mathcal{A}_{II}$$





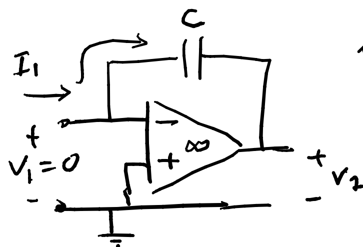
$I_1 = -I_2$

$V_1 = V_2 - RI_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = y_I \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$y_I = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

$$y = y_I \cdot y_{II}$$
$$= \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -sC & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -sRC & 0 \\ -sC & 0 \end{bmatrix}$$



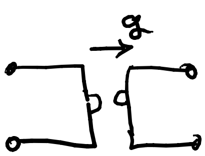
$sC\, V_2 = -I_1$

$V_1 = 0$

$I_1 = -sC\, V_2$

$$y_{II} = \begin{bmatrix} 0 & 0 \\ -sC & 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -sRC & 0 \\ -sC & 0 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \Longrightarrow V_2 = \frac{-1}{sRC} V_1 \Rightarrow \frac{V_2}{V_1} = \frac{-1}{sRC}$$



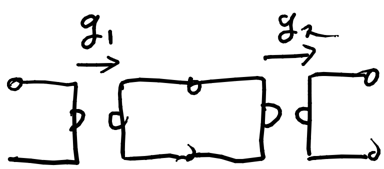
$I_1 = -g V_2$

$I_2 = g V_1$

$V_1 = \left(\frac{-1}{g}\right)(-I_2)$

$I_1 = -g V_2$

$$y_{gyr} = \begin{bmatrix} 0 & -1/g \\ -g & 0 \end{bmatrix}$$



$$y = y_I \cdot y_{II} = \begin{bmatrix} 0 & -1/g_1 \\ -g_1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1/g_2 \\ -g_2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} g_2/g_1 & 0 \\ 0 & g_1/g_2 \end{bmatrix}$$

$V_1 = \frac{g_2}{g_1} V_2$, $I_1 = \frac{g_1}{g_2}(-I_2)$

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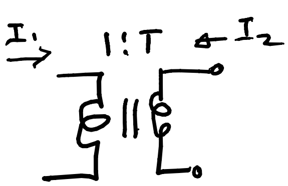
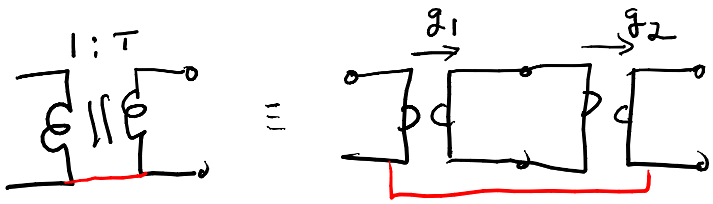


Σ amp turns $= 0$ $\quad I_1(1) + I_2(T) = 0 \Rightarrow I_1 = -T \cdot I_2$

$\Rightarrow T = g_1/g_2$

$V_2 = T \cdot V_1 = \frac{g_1}{g_2} V_1$

also gives $T = g_1/g_2$



$T = g_1/g_2$

$V_2 = T V_1$

$I_1 = -T I_2$

no 2-port Z or Y for the ideal transformer