1)

a) the current-voltage relationship for c_1 is

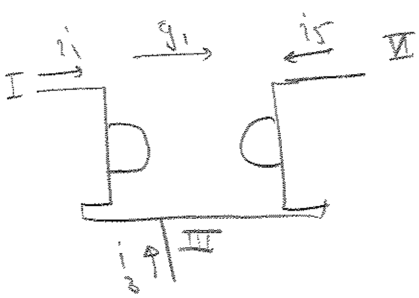
$$i_3 = c_1 s v_3 - c_1 s v_6$$
$$i_6 = c_1 s v_6 - c_1 s v_3$$

$$\Rightarrow Y_{ind\,c_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sc_1 & 0 & 0 & -sc_1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -sc_1 & 0 & 0 & sc_1 \end{bmatrix}$$

For c_2:
$$i_5 = c_2 s v_5 - c_2 s v_6$$
$$i_6 = c_2 s v_6 - c_2 s v_5$$

$$\Rightarrow Y_{ind\,c_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 s & 0 & -c_2 s \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_2 s & 0 & 0 & c_2 s \end{bmatrix}$$

For gyrator we have the following relationship:



$$i_3 = g_1(v_5 - v_1)$$
$$i_1 = -g_1(v_5 - v_3)$$
$$i_5 = g_1(v_1 - v_3)$$

so $$Y_{ind\,g_1} = \begin{bmatrix} 0 & 0 & g_1 & 0 & -g_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -g_1 & 0 & 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ g_1 & 0 & -g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) ground node 6 so $V_6 = 0$

$$Y_{nodal} = \begin{bmatrix} 0 & 0 & g_1 & 0 & -g_1 \\ 0 & 0 & 0 & -g_2 & g_2 \\ -g_1 & 0 & sc_1 & 0 & g_1 \\ 0 & g_2 & 0 & sc_2 & -g_2 \\ g_1 & -g_2 & -g_1 & g_2 & 0 \end{bmatrix}$$

c) eliminate node 3 first $\Longrightarrow i_3 = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = Y_{nodal} \times \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} \qquad \text{(I)}$$

so $i_3 = -g_1 v_1 + sc_1 v_3 + g_1 v_5$

$i_3 = 0 \Longrightarrow -g_1 v_1 + g_1 v_5 + sc_1 v_3 = 0 \Longrightarrow v_3 = \begin{bmatrix} \frac{g_1}{sc_1} & \frac{-g_1}{sc_1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_5 \end{bmatrix}$

from equation (I) we have:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} g_1 \\ 0 \end{bmatrix} \cdot v_3 + \begin{bmatrix} 0 & -g_1 \\ -g_2 & g_2 \end{bmatrix} \begin{bmatrix} v_4 \\ v_5 \end{bmatrix}$$

$$\begin{bmatrix} i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & g_2 \\ g_1 & -g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -g_1 \end{bmatrix} \cdot v_3 + \begin{bmatrix} sc_2 & -g_2 \\ g_2 & 0 \end{bmatrix} \begin{bmatrix} v_4 \\ v_5 \end{bmatrix}$$

Use the relationship for v_3 $\Longrightarrow$

$$\begin{cases} i_1 = -g_1 v_5 - \frac{g_1^2}{sc_1} v_5 + \frac{g_1^2}{sc_1} v_1 \\ i_2 = -g_2 v_4 + g_2 v_5 \\ i_5 = g_1 v_1 - g_2 v_2 - \frac{g_1^2}{sc_1} v_1 + \frac{g_1^2}{sc_1} v_5 + g_2 v_4 \\ i_4 = sc_2 v_4 + g_2 v_2 - g_2 v_5 \end{cases}$$

$i_3 = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} \frac{g_1^2}{sc_1} & 0 & 0 & -g_1 - \frac{g_1^2}{sc_1} \\ 0 & 0 & -g_2 & g_2 \\ 0 & g_2 & sc_2 & -g_2 \\ g_1 - \frac{g_1^2}{sc_1} & -g_2 & g_2 & \frac{g_1^2}{sc_1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_4 \\ v_5 \end{bmatrix}$$

let $i_4 = 0$

$i_4 = g_2 v_2 + sc_2 v_4 - g_2 v_5 = 0 \Longrightarrow v_4 = -\frac{g_2}{sc_2} v_2 + \frac{g_2}{sc_2} v_5$

then
$$\begin{cases} i_2 = +\frac{g_2^2}{sc_2} v_2 + g_2 v_5 - \frac{g_2^2}{sc_2} v_5 \\ i_5 = g_1 v_1 - \frac{g_1^2}{sc_1} v_1 - g_2 v_2 - \frac{g_2^2}{sc_2} v_2 + \frac{g_2^2}{sc_2} v_5 + \frac{g_1^2}{sc_1} v_5 \\ i_4 = 0 \end{cases}$$

$$\Longrightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_5 \end{bmatrix} = \begin{bmatrix} \frac{g_1^2}{sc_1} & 0 & -g_1 - \frac{g_1^2}{sc_1} \\ 0 & \frac{g_2^2}{sc_2} & g_2 - \frac{g_2^2}{sc_2} \\ g_1 - \frac{g_1^2}{sc_1} & -g_2 - \frac{g_2^2}{sc_2} & \frac{g_1^2}{sc_1} + \frac{g_2^2}{sc_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_5 \end{bmatrix}$$

let $i_5 = 0$ then $-\left(g_2 + \frac{g_2^2}{sc_2}\right)v_2 + \left(g_1 - \frac{g_1^2}{sc_1}\right)v_1 + \left(\frac{g_1^2}{sc_1} + \frac{g_2^2}{sc_2}\right)v_5 = 0$

$$\Longrightarrow v_5 = \left(\frac{g_1^2 c_2 - g_1 s c_1 c_2}{g_1^2 c_2 + g_2^2 c_1}\right)v_1 + \left(\frac{g_2 s c_1 c_2 + g_2^2 c_1}{g_1^2 c_2 + g_2^2 c_1}\right)v_2$$

So $$i_1 = \frac{g_1^2}{sc_1}v_1 + \left(\frac{g_1^2 c_2 - g_1 s c_1 c_2}{g_1^2 c_2 + g_2^2 c_1}\right)\left(-g_1 - \frac{g_1^2}{sc_1}\right)v_1 + \left(\frac{g_2 s c_1 c_2 + g_2^2 c_1}{g_1^2 c_2 + g_2^2 c_1}\right)\left(-g_1 - \frac{g_1^2}{sc_1}\right)$$

$$i_2 = \left(\frac{g_1^2 c_2 - g_1 s c_1 c_2}{g_1^2 c_2 + g_2^2 c_1}\right)\left(g_2 - \frac{g_2^2}{sc_2}\right)v_1 + \frac{g_2^2}{sc_2}v_2 + \left(\frac{g_2 s c_1 c_2 + g_2^2 c_1}{g_1^2 c_2 + g_2^2 c_1}\right)\left(g_2 - \frac{g_2^2}{sc_2}\right)v_2$$

from the equations above for i_1 and i_2 we have the relationship between $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ and $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

c) eliminate all three nodes at the same time:

From equation (I)

y_{11} $\quad$ y_{12}

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & g_1 & 0 & -g_1 \\ 0 & 0 & 0 & -g_2 & g_2 \\ -g_1 & 0 & sc_1 & 0 & g_1 \\ 0 & g_2 & 0 & sc_2 & -g_2 \\ g_1 & -g_2 & -g_1 & g_2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

y_{21} $\quad$ y_{22}

$$\Longrightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = y_{11}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + y_{12}\left(-y_{22}^{-1} \cdot y_{21}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \left(y_{11} - y_{12}\, y_{22}^{-1}\, y_{21}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix} = y_{21} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + y_{22} \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \underbrace{\frac{(c_1 g_2^2 + g_1^2 c_2)(g_2^2 + s^2 c_1 c_2)}{s}}_{\mu_{11}} & \underbrace{\frac{(c_1 g_2^2 + g_1^2 c_2)(g_1 + s c_2)(g_2 + s c_2)(-g_1 g_2)}{s}}_{\mu_{12}} \\ \underbrace{\frac{(c_1 g_2^2 + g_1^2 c_2)(-g_2 + s c_2)(-g_1 + s c_1)(-g_1 g_2)}{s}}_{\mu_{21}} & \underbrace{\frac{(c_1 g_2^2 + g_1^2 c_2) g_2^2 (g_1^2 + s^2 c_1 c_2)}{s}}_{\mu_{21}} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The method which eliminates all the nodes together is easier and faster.

d) from above:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$V_2 = R I_2$ R: load Resistance

$$i_1 = \mu_{11} v_1 + \mu_{12} v_2$$

$$i_2 = \mu_{21} v_1 + \mu_{22} v_2 \Longrightarrow R i_2 = R \mu_{21} v_1 + \mu_{22} R v_2$$

then $v_2 = R \mu_{21} v_1 + R \mu_{22} v_2 \Longrightarrow R \mu_{21} v_1 = (1 - R\mu_{22}) v_2 \Longrightarrow v_2 = \overbrace{\frac{R\mu_{21}}{1 - R\mu_{22}}}^{S} v_1$

so $V_2 = S v_1 \Longrightarrow i_1 = (\mu_{11} + \mu_{12} S) v_1 \Longrightarrow \frac{i_1}{v_1} = \mu_{12} S + \mu_{11}$

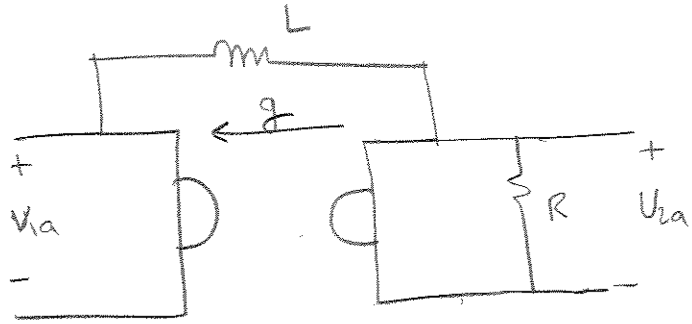
so $Z_{in} = \mu_{12} \left(\frac{R\mu_{21}}{1 - R\mu_{22}} \right) + \mu_{11}$

Hw #3 ENEE610

(7)

Problem 2) a) the adjoint of a gyrator is a gyrator with negative gyration.

adjoint:



$$\Delta(T) = V_{int}^{T}\, \Delta Y^{T}\, V_{inta}$$

voltage transfer function

$, V_{int} = [V_1 \quad V_2]$

$V_{inta} = [V_{1a} \quad V_{2a}]$

finding Y:

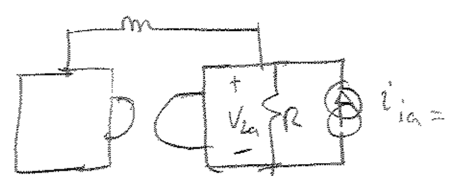
$$Y = Y_R + Y_L + Y_g \qquad Y_L = \begin{bmatrix} \frac{1}{Ls} & \frac{-1}{Ls} \\ \frac{-1}{Ls} & \frac{1}{Ls} \end{bmatrix} \qquad Y_R = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} \qquad Y_g = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} \frac{1}{Ls} & \frac{-1}{Ls} - g \\ \frac{-1}{Ls} + g & \frac{1}{Ls} + G \end{bmatrix} \qquad G = \frac{1}{R}$$

derivative w/respect to R

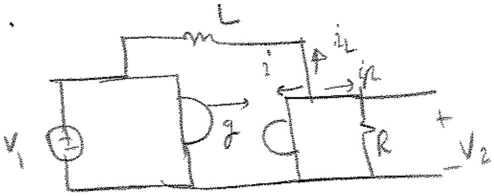
$$\Delta Y = \begin{bmatrix} 0 & 0 \\ 0 & \Delta G \end{bmatrix} \qquad \frac{\Delta(T)}{\Delta G} = V_2 \cdot V_{2a}$$

to find derivative w/respect to R we have:



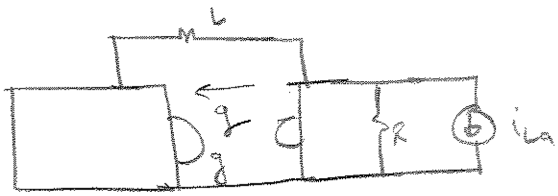
$$\begin{cases} V_{1a} = 0 \\ i_{2a} = 1 \end{cases}$$

find V_2:



$$i_R + i_L + i = 0 \Longrightarrow \frac{V_2}{R} + gV_1 + \frac{1}{Ls}V_2 - \frac{1}{Ls}V_1 = 0 \Longrightarrow V_2 = \left(\frac{1-sLg}{1+sLG}\right)V_1$$

find V_{2a}:



$$Z^{-1} = \frac{1}{Ls} + \frac{1}{R} = \frac{1}{Ls} + G = \frac{sLG+1}{Ls} \Longrightarrow V_{2a} = Z \cdot i_{La} = \frac{-Ls}{1+sLG}$$

$$\frac{\partial T}{\partial G} = V_2 \cdot V_{2a} = \left(\frac{1-sLG}{sLG+1}\right)\left(\frac{-Ls}{1+sLG}\right) = \frac{L^2 g s^2 - Ls}{(1+sLG)^2}$$

With respect to L:

$$\Delta T = V_{int}^T \, \Delta Y \, V_{int\text{-}a}$$

$$T = \frac{V_2}{V_1} \qquad \Delta Y = \begin{bmatrix} \frac{1}{\Delta Ls} & -\frac{1}{\Delta Ls} \\ -\frac{1}{\Delta Ls} & \frac{1}{\Delta Ls} \end{bmatrix}$$

$$V_{int_a} = [V_{1a} \quad V_{2a}]^T = [0 \quad V_{2a}]^T = \begin{bmatrix} 0 \\ V_{2a} \end{bmatrix}$$

$V_{1a} = 0$, $i_{L_a} = 1$

$$\Delta T = [U_1 \quad U_2] \begin{bmatrix} \frac{1}{\Delta Ls} & -\frac{1}{\Delta Ls} \\ -\frac{1}{\Delta Ls} & \frac{1}{\Delta Ls} \end{bmatrix} [0 \quad V_{2a}]^T = \frac{V_{2a}(V_2 - V_1)}{\Delta Ls}$$

assume $\lambda = \frac{1}{L}$ $\Longrightarrow \Delta Y = \begin{bmatrix} \frac{\Delta\lambda}{s} & -\frac{\Delta\lambda}{s} \\ -\frac{\Delta\lambda}{s} & \frac{\Delta\lambda}{s} \end{bmatrix}$

$$\Longrightarrow \Delta V_2 = \begin{bmatrix} V_1 & V_2 \end{bmatrix} \begin{bmatrix} \frac{\Delta\lambda}{s} & -\frac{\Delta\lambda}{s} \\ -\frac{\Delta\lambda}{s} & \frac{\Delta\lambda}{s} \end{bmatrix} \begin{bmatrix} 0 \\ V_{2a} \end{bmatrix}$$

$$\Longrightarrow \Delta V_2 = V_{2a} \, (V_2 - V_1) \, \frac{\Delta\lambda}{s} \qquad \Longrightarrow V_2 = H(s) V_1$$

$$\Longrightarrow \Delta V_2 = V_{2a} \left(H(s) V_1 - V_1 \right) \frac{\Delta\lambda}{s} \qquad \Longrightarrow \Delta\left(\frac{V_2}{V_1}\right) = V_{2a} \left(H(s) - 1 \right) \frac{\Delta\lambda}{s}$$

$$\Longrightarrow \frac{\Delta\left(\frac{V_2}{V_1}\right)}{\Delta\lambda} = \frac{V_{2a}}{s} \left(H(s) - 1 \right)$$

we know from before $H(s) = \frac{1 - sLg}{sLG + 1}$ and $V_{2a} = \frac{-Ls}{sLG + 1}$

substituting

$$\Longrightarrow \frac{\Delta\left(\frac{V_2}{V_1}\right)}{\Delta\lambda} = \frac{L^2 s}{(1 + sLG)^2} (g + G)$$

$$\Delta\lambda = \frac{-1}{L^2} \Delta L \qquad \Longrightarrow \frac{\Delta\left(\frac{V_2}{V_1}\right)}{\frac{-1}{L^2} \Delta L} = \frac{L^2 s}{(1 + sLG)^2} (g + G)$$

$$\Longrightarrow \frac{\Delta\left(\frac{V_2}{V_1}\right)}{\Delta L} = \frac{-s}{(1 + sLG)^2} (g + G)$$

Direct derivation with respect to R and L

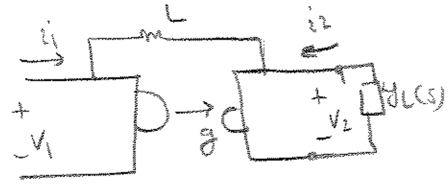
$$\frac{V_2}{V_1} = \frac{1 - sLg}{1 + sLG} \qquad \frac{\partial\left(\frac{V_2}{V_1}\right)}{\partial G} = \frac{-sL(1 - sLg)}{(1 + sLG)^2} = \frac{-sL + L^2 g s^2}{(1 + sLG)^2}$$

the same

(b)

$$\frac{\partial\left(\frac{V_2}{V_1}\right)}{\partial L} = \frac{-sg - sG}{(1+sLG)^2}$$ the same

b)



$i_2 = y_L(s) V_2$

From class notes:

$$y(s) = \frac{i_1}{V_1} = \frac{1}{Ls} - \left(-\frac{1}{Ls} - g\right)\left(y_L + \frac{1}{Ls}\right)^{-1}\left(g - \frac{1}{Ls}\right)$$

$$\Longrightarrow y(s) = \frac{g^2 L - \frac{y_L}{s}}{L \cdot y_L + 1/s} \Longrightarrow y(s) = \frac{Y_L + sg^2 L}{sY_L L + 1} \Longrightarrow Y_L(s) = \frac{y(s) - sg^2 L}{1 - sLy(s)}$$

in order to check the input admittance for $y_L = G$

$$\left\{\begin{array}{l} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Ls} & -\frac{1}{Ls} - g \\ -\frac{1}{Ls} + g & \frac{1}{Ls} + G \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \\ i_2 = -\frac{1}{R} V_2 = -GV_2 \end{array}\right. \Longrightarrow \frac{i_1}{V_1} = y(s) = \frac{\frac{1}{Ls}(V_1) + \left(-\frac{1}{Ls} - g\right)\left(-\frac{1}{Ls} + g\right)/\left(\frac{1}{Ls} + 2G\right)}{V_2}$$

$$\Longrightarrow \frac{i_1}{V_1} = \frac{sg^2 L + G}{sLG + 1}$$

d) $$Ev\{y(s)\} = \frac{1}{2}\left(y(s) + y(-s)\right) = \frac{1}{2}\left(\frac{y_L(s) + Lsg^2}{1 + Lsy_L(s)} + \frac{y_L(-s) - Lsg^2}{1 - Lsy_L(-s)}\right)$$

$$\Longrightarrow Ev\{y(s)\} = \frac{(y_L(s) + y_L(-s)) - L^2 s^2 g^2 (y_L(s) + y_L(-s))}{(\ldots)(\ldots)} \Rightarrow$$

$$2\ EV\{y(s)\} = \frac{(y_L(s)+y_L(-s))(1-L^2s^2g^2)}{(1+Lsy_L(s))[1-Ls\,y_L(-s)]}$$

$$EV(y(s)) = \frac{1}{2}\ \frac{[y_L(s)+y_L(-s)][1-L^2s^2g^2]}{[1+Lsy_L(s)][1-Lsy_L(-s)]}$$

$y_L(s)=G$, $y_L(-s)=G$

$$\Longrightarrow EV\{y(s)\} = \frac{1}{2}\ \frac{2G(1-L^2s^2g^2)}{(1+LsG)(1-LsG)} = \frac{G(1-L^2s^2g^2)}{(1-L^2s^2G^2)}$$

$R=5$, $g=3$, $L=2$ $\qquad G(1-L^2s^2g^2)=0 \Longrightarrow 1-36s^2=0 \Longrightarrow s=\pm 1/6$ = zeros