HW #1 ENEE 610 Sanaz Adl (1)

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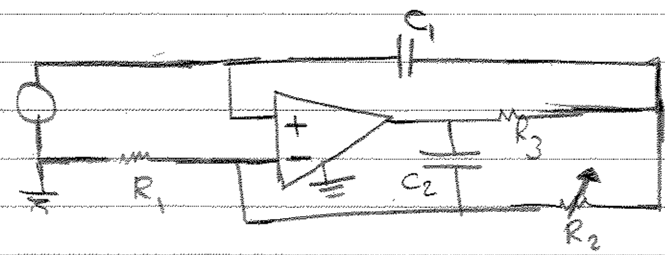
(2) (a), (b)



KCL:

$$i_1 + i_6 - i_7 = 0$$

$$i_2 + i_3 + i_6 = 0$$

$$i_3 + i_5 + i_8 = 0$$

$$i_4 - i_5 - i_6 + i_7 - i_8 = 0$$

KVL:

$$-v_2 - v_3 + v_4 + v_5 = 0$$

$$-v_1 - v_2 + v_4 + v_6 = 0$$

$$v_1 - v_4 + v_7 = 0$$

$$-v_3 + v_4 + v_8 = 0$$

a; $v = \mathcal{B} i$ so: device equations are as follows:

$sC_1 v_1 = i_1$ $\quad$ $v_5 = k v_4$

$sC_2 v_2 = i_2$ $\quad$ $G_3 v_6 = i_6$

$v_3 = v + 0.i_3$ $\quad$ $G_2 v_7 = i_7$

$i_3 = -i$ $\quad$ $G_1 v_8 = i_8$

$i_4 = 0$

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So from the above device equations.

$$\begin{bmatrix} sC_1 &&&&&&& \\ & sC_2 &&&&&& \\ && 1 &&&&& \\ &&& 0 &&&& \\ &&& K & -1 &&& \\ &&&&& G_3 && \\ &&&&&& G_2 & \\ &&&&&&& G_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -v \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 0 &&&&& \\ &&& 1 &&&& \\ &&&& 0 &&& \\ &&&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix}$$

from KCL:

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 \end{array}\right] i_b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad C = \text{cutset Matrix} = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 \end{array}\right]$$

from KVL:

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] v_b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad T = \left[\begin{array}{cccc|cccc} 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{array}\right.$$

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from KCL:

$$i_t = \begin{bmatrix} 0 & -1 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix} i_l$$

from KVL

$$v_l = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} v_t$$

$$a\begin{bmatrix} v_t \\ v_l \end{bmatrix} - ae = B\begin{bmatrix} i_t \\ i_l \end{bmatrix} \Rightarrow a\begin{bmatrix} I_t \\ K_t \end{bmatrix} v_t - e = B\begin{bmatrix} -K_t^T \\ I_l \end{bmatrix} i_l$$

$$0 = i^T v = i^T \begin{bmatrix} v_t \\ K_t v_t \end{bmatrix} = i_l^T \left[K_l^T + K_t \right] v_t$$

d, e, f

$$-v_2 - v_3 + (k+1)v_4 = 0 \qquad \text{if } k \neq -1,\ v_4 = \frac{1}{k+1}(v_2 + v_3)$$

$$v_3 = v$$

$$G_1 v_3 - G_1 v_4 = G_1 v - \frac{G_1}{1+k}(v_2 + v) = i_8$$

$$i_6 = G_3(v_1 + v_2 - v_4) = G_3\left(v_1 + v_2 - \frac{1}{1+k}[v_2 + v_3]\right)$$

$$i_7 = G_2(-v_1 + v_4) = G_2\left(-v_1 + \frac{1}{1+k}[v_2 + v_3]\right)$$

$$i_5 + i_6 - i_7 + i_8 = 0 \Rightarrow i_5 = i_6 - i_7 - i_8 = -G_3\left(v_1 + v_2 - \frac{1}{1+k}[v_2 + v]\right)$$

$$- G_2\left(-v_1 + \frac{1}{1+k}[v_2 + v]\right) + G_1 v - \frac{G_1}{1+k}[v_2 + v]$$

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(1) $sC_1 v_1 = -i_6 + i_7 = -G_3\left(v_1 + v_2 - \frac{1}{1+k}[v_2+v]\right) + G_2\left(-v_1 + \frac{1}{1+k}[v_2+v]\right)$

(2) $sC_2 v_2 = -i_5 - i_6 = -[-i_6 + i_7 - i_8] - i_6 = -i_7 + i_8$

$$= -G_2\left[-v_1 + \frac{1}{1+k}[v_2+v]\right) + G_1\left(v - \frac{1}{1+k}[v_2+v]\right)$$

$\Rightarrow sC_2 v_2 = G_2 v_1 + [-G_2 - G_1]\frac{1}{1+k} v_2 + (-G_2 - G_1)\frac{1}{1+k} v + G_1 v$

$i = -i_3 = -(-i_5 - i_8) = -i_6 - i_7 - i_8 + i_8 = -i_6 - i_7$

$$= -G_3\left(v_1 + v_2 - \frac{1}{1+k}[v_2+v]\right) - G_2\left(-v_1 + \frac{1}{1+k}[v_2+v]\right)$$

So $\Rightarrow$

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} s \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -(G_2+G_3) & -G_3 + \frac{G_2+G_3}{1+k} \\ G_2 & \frac{-G_1-G_2}{1+k} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{G_2+G_3}{1+k} \\ G_1 + \frac{G_1-G_2}{1+k} \end{bmatrix} v$$

$$i = \begin{bmatrix} G_2 - G_3 & -G_3 + \frac{G_3-G_2}{1+k} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{G_3-G_2}{1+k} \end{bmatrix} v$$

$$\Rightarrow Y = \begin{bmatrix} \frac{G_3-G_2}{1+k} \end{bmatrix} + \begin{bmatrix} G_2 - G_3 & -G_3 + \frac{G_3-G_2}{1+k} \end{bmatrix} \begin{bmatrix} sC_1 + (G_2+G_3) & G_3 - \frac{G_2+G_3}{1+k} \\ -G_2 & sC_2 + \frac{G_2+G_1}{1+k} \end{bmatrix}^{-1} \begin{bmatrix} \frac{G_2+G_3}{1+k} \\ G_1 - \frac{G_2+G_1}{1+k} \end{bmatrix}$$

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if $k \to \infty$ then $Y \approx \begin{bmatrix} G_2 - G_3 & -G_3 \end{bmatrix} \begin{bmatrix} sC_1 + (G_2+G_3) & G_3 \\ -G_2 & sC_2 \end{bmatrix} \begin{bmatrix} 0 \\ G_1 \end{bmatrix}$

$$Y = \begin{bmatrix} G_2 - G_3 & -G_3 \end{bmatrix} \begin{bmatrix} -G_3 G_1 \\ (sC_1 + (G_2+G_3))G_1 \end{bmatrix} \frac{1}{s^2 C_1 C_2 + sC_2 (G_2+G_3) + G_2 G_3}$$

for $C_1 \gg C_2$, $G_3 \gg G_1 \sim G_2$ $\Rightarrow Y = -G_3 \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -G_3 G_1 \\ sC_1 G_1 \end{bmatrix} \frac{1}{-G_2 G_3}$

$$\approx \frac{G_1}{G_2} s C_1$$