File: c:\temp\courses\fall2005\610\final_exam.doc RWN 12/13/05 corrected printout 12/13-14b/05
ENEE 610 Final Exam; 12/19/05
Open book \& notes. Only signed work, which designates all the work is your own, will be graded. 1. (50 points)

For the following small signal version of a Colpitts oscillator, use the BJT transistor equivalent circuit shown on the right.

a. Draw an oriented graph numbering branches as per the element numbering and nodes $0=\mathrm{e}=$ ground, $1=\mathrm{b}, 2=\mathrm{c}$; orient all branches to point toward the lower numbered node. For this include C 1 and r 1 together in branch 1 , so $\mathrm{y} 1(\mathrm{~s})=\mathrm{g} 1+\mathrm{sC} 1$, and C 2 and r 2 in branch 2 as $\mathrm{y} 2(\mathrm{~s})=\mathrm{g} 2+\mathrm{sC} 2$.
b. Choose branches 1 and 2 for the tree and give the cut set and tie set matrices.
c. Give the branch by branch admittance and use the cut set matrix to find the $(2 \times 2)$ nodal admittance matrix, $Y(s)$. Check by finding the (3x3) indefinite admittance Yind(s) and then grounding the e node (chosen as node 3 prior to becoming 0 when grounding).
d. Give the even and odd parts of $\operatorname{det}[\mathrm{Y}(\mathrm{s})]$. For the circuit to be an oscillator both of these should be 0 at the oscillation frequency $\mathrm{s}=\mathrm{j} \omega_{0}$.
e. The two equations of part d determine a bias collector current, Ic, since $\mathrm{gm}=\mathrm{Ic} / \mathrm{V}_{\mathrm{T}}, \mathrm{g} 1=(1 / \mathrm{r} 1)=\mathrm{g}_{\pi}=\mathrm{Ic} /\left(\alpha \mathrm{V}_{\mathrm{T}}\right), \mathrm{g} 2=(1 / \mathrm{r} 2)=\mathrm{g}_{0}=\mathrm{Ic} / \mathrm{V}_{\mathrm{A}}$. Determine Ic in terms of $\alpha, \mathrm{VT}$, VA, $\mathrm{L}, \mathrm{C} 1$ and C 2 . [the normal $\mathrm{g}_{\pi}=\mathrm{Ic} /\left(\{\beta+1\} \mathrm{V}_{\mathrm{T}}\right)$ but for this problem Ic/( $\left.\alpha \mathrm{V}_{\mathrm{T}}\right)$ is used ].
f. Choose $\alpha=0.999, \mathrm{~V}_{\mathrm{A}}=100, \mathrm{~V}_{\mathrm{T}}=0.026, \mathrm{~L} 3=5 \mathrm{nHy}, \mathrm{C} 1=5 \mathrm{pFd}$ with $0.01<(\mathrm{C} 2 / \mathrm{C} 1)<100$, and make reasonable approximations to determine for which C 2 the circuit will oscillate and find Ic and $\omega_{0}$ for those C2.
2. (50 points)
a. For what values of the real parameters $a$ and $b$ is the admittance $\mathrm{y}(\mathrm{s})=\left[\left(\mathrm{s}^{2}+\mathrm{a}\right)\left(\mathrm{s}^{2}+\mathrm{b}\right) /\left[\mathrm{as}\left(\mathrm{s}^{2}+1\right)\right]\right.$ positive-real?
b. For $b=1 / a$ determine for which a this $y(s)$ is lossless and give a second Foster synthesis with a as a parameter. Give also a first Cauer synthesis of this $y(s)$ when $b=1 / a$
c. Using the second Foster expansion show that any rational lossless driving point admittance satisfies $y_{L C}(\mathrm{~s})=\mathrm{sf}\left(\mathrm{s}^{2}\right)$ and exhibit $\mathrm{f}(\mathrm{s})\left[\right.$ not $\mathrm{f}\left(\mathrm{s}^{2}\right)$ ] as a partial fraction expansion. In a lossless second Foster synthesis of $y_{L C}(s)$, if each inductor is replaced by a resistor then we obtain an $R C$ second Foster synthesis of $y_{R C}(s)$. Give $y_{R C}(s)$ in terms of $\mathrm{f}(\mathrm{s})$ and from that give a partial fraction type expansion for $\mathrm{y}_{\mathrm{RC}}(\mathrm{s})$ and then in turn an RC synthesis.
d. Give the necessary and sufficient conditions that a rational $y(s)$ (with real coefficients) is the driving point admittance of an RC circuit.
e. For such $\operatorname{y~}_{\mathrm{RC}}(\mathrm{s})$ determine the location of its even part, $\operatorname{Ev}\left[\mathrm{y}_{\mathrm{RC}}(\mathrm{s})\right]$, poles and the relation to them of the even part zeros. Sketch $\operatorname{Ev}\left[\mathrm{y}_{\mathrm{RC}}(\mathrm{s})\right]$ for real $\mathrm{s}\{$ hint; look at dEv[y]/ds\}

