file: e:/courses/fall2005/610/even_part.mcd   RWN   10/08-13/05

$j := \sqrt{-1}$

set k, a zero of the even part

$kr := 1$ $\quad ki := 2$ $\quad k := kr + j \cdot ki$ $\quad k = 1 + 2j$ $\quad (|k|)^2 = 5$

Set the numerator of the even part of a degree two function as

$$Evnum(s,k) := (s-k)\cdot(s-k)\cdot(s+k)\cdot\left(s+\overline{k}\right)$$

$$Evnum(s,k) := \left[s^2 - \left(k+\overline{k}\right)\cdot s + (|k|)^2\right]\cdot\left[s^2 + \left(k+\overline{k}\right) s + (|k|)^2\right]$$

$$Evnum(s,k) := s^4 + \left[2\cdot(|k|)^2 - \left(k+\overline{k}\right)^2\right]\cdot s^2 + (|k|)^4$$

$Evnum(k,k) = -3.553\cdot 10^{-15}$

The coefficients as functions of k are

$e2(k) := \left[2\cdot(|k|)^2 - \left(k+\overline{k}\right)^2\right]$ $\qquad e2(k) = 6$

$eo(k) := (|k|)^4$ $\qquad eo(k) = 25$

Define the degree two admittance as y(s) with default coefficients

$a := 1$ $\quad b := 1$ $\quad c := 1$ $\quad d := 1$

$$y(s) := \frac{\left(s^2 + a\cdot s + b\right)}{\left(s^2 + c\cdot s + d\right)}$$

$$2\cdot Ev(y(s) := y(s) + y(-s)$$

$$numEv\left[y(s) := Ev\left(s^2 + a\cdot s + b\right)\left(s^2 - c\cdot s + d\right) := s^4 + (b + d - ac)\cdot s^2 + b\cdot d\right]$$

As there are four parameters and two coefficients to be set in the even part we set two, here a and b, and calculate the remainder as functions of k

$a := 2$ $\quad b := 4$

$d(k) := \dfrac{eo(k)}{b}$ $\qquad d(k) = 6.25$

$c(k) := \dfrac{(b + d(k) - e2(k))}{a}$ $\qquad c(k) = 2.125$

This allows calculation of the values and functions needed for a cascade synthesis

$$y(k) := \frac{\left(k^2 + a\cdot k + b\right)}{\left(k^2 + c(k)\cdot k + d(k)\right)}$$

$k = 1 + 2j$

$y(k) = 0.847 + 0.188j$

$g1(k) := y(k)$ $\qquad g1(k) = 0.847 + 0.188j$

$C1(k) := \dfrac{y(k)}{k}$ $\qquad C1(k) = 0.245 - 0.301j$

The load admittance is found by using the appropriate Richards function

$$yL(s) := y(k)\cdot\frac{(s\cdot y(s) - k\cdot y(k))}{(s\cdot y(k) - k\cdot y(s))}$$

Factoring this and dividing out (s-k)(s+k)

To form an accurate yL(s) the coefficients, a's, in the numerator and those, b's, in the denominator are found and then long division used to factor

$k\cdot y(k) = 0.471 + 1.882j$

$a2L(k) := a - k\cdot y(k)$ $\quad a2L(k) = 1.529 - 1.882j$

$a1L(k) := b - c(k)\cdot k\cdot y(k)$ $\quad a1L(k) = 3 - 4j$

$a0L(k) := d(k)\cdot k\cdot y(k)$ $\quad a0L(k) = 2.941 + 11.765j$

$b3L(k) := y(k)$ $\quad b3L(k) = 0.847 + 0.188j$

$b2L(k) := c(k)\cdot y(k) - k$ $\quad b2L(k) = 0.8 - 1.6j$

$b1L(k) := d(k)\cdot y(k) - a\cdot k$ $\quad b1L(k) = 3.294 - 2.824j$

$b0L(k) := b\cdot k$ $\quad b0L(k) = 4 + 8j$

Long division of s^2-k^2 $\quad k^2 = -3 + 4j$

$$numL(s) := (3 - 4j)\cdot\frac{\left[s^3 + (1.529 - 1.882j)\cdot s^2 + (3 - 4j)\cdot s + (2.941 + 11.765j)\right]}{s^2 - (-3 + 4j)}$$

as

$$k^2\cdot(a - k\cdot y(k)) = 2.941 + 11.765j$$

$$numyL(s) := y(k)\ (s + a2L(k))$$

$$denyL(s) := (y(k)\cdot s + b2L(k))$$

$k^2\cdot y(k) + b1L(k) = 1.332j\cdot 10^{-15}$ $\quad k^2\cdot b2L(k) = 4 + 8j$

The end result of the long division is

$$yL(s) := y(k)\cdot\frac{(s + a2L(k))}{(y(k)\cdot s + b2L(k))}$$

$$yL\left(\overline{k}\right) = 1.059 + 0.235j$$

Form the gyration conductance and capacitance for the second stage of the cascade

$g2(k) := yL\left(\overline{k}\right)$ $\quad g2(k) = 1.059 + 0.235j$

$C2(k) := \dfrac{yL\left(\overline{k}\right)}{\overline{k}}$ $\quad C2(k) = 0.118 + 0.471j$

Finally form the load admittance for the second stage using the Richards' function on yL(s) at the conjugate of k

$$yLL(s) := yL(\overline{k})\cdot\frac{\left(s\cdot yL(s) - \overline{k}\cdot yL(\overline{k})\right)}{\left(s\cdot yL(\overline{k}) - \overline{k}\cdot yL(s)\right)}$$

This is a constant as expected since it evaluates as 1

$yLL(1) = 1$ $\quad yLL(2) = 1$

But to exactly form the numerator and denominator before the cancellations form the coefficients of both

$a2LL(k) := y(k)$ $\quad y(k) = 0.847 + 0.188j$

$a1LL(k) := y(k)\cdot a2L(k) - \overline{k}\cdot yL(\overline{k})\cdot y(k)$ $\quad yL(\overline{k}) = 1.059 + 0.235j$

$a0LL(k) := b2L(k)$ $\quad a1LL(k) = 0$

$b2LL(k) := y(k)\cdot yL(\overline{k})$ $\quad b2LL(k) = 0.853 + 0.399j$

$b1LL(k) := yL(\overline{k})\cdot b2L(k) - \overline{k}\cdot y(k)$ $\quad b1LL(k) = 0$

$b0LL(k) := -\overline{k}\cdot a2L(k)$ $\quad a0LL(k) = 0.8 - 1.6j$

$$yLL(s) := yL(\overline{k})\cdot\frac{\left[y(k)\cdot s^2 + s\cdot\left(y(k)\cdot a2L(k) - \overline{k}\cdot y(k)\cdot yL(\overline{k})\right) - \overline{k}\cdot yL(\overline{k})\cdot b2L(k)\right]}{\left[yL(\overline{k})\cdot y(k)\, s^2 + s\cdot\left(yL(\overline{k})\cdot b2L(k) - \overline{k}\cdot y(k)\right) - \overline{k}\cdot y(k)\cdot a2L(k)\right]}$$

$yLL(1) = 1$ $\quad yLL(2) = 1$ $\quad yLL(3) = 1$

Next calculate the coefficients in the resulting 2-port Y(s) on combining the two cascade 2-ports into one 2-port between input and load yLL=1

$$C(k) := \frac{(C1(k)\cdot C2(k))}{(C1(k) + C2(k))}$$ $\quad C(k) = 0.471$

$$g(k) := \frac{(g1(k)\cdot C2(k) + g2(k)\cdot C1(k))}{(C1(k) + C2(k)}$$ $\quad g(k) = 0.941$

$$L(k) := \frac{\left(g1(k)^2\right)}{(C1(k) + C2(k))}$$ $\quad L(k) = 1.882$

$$T(k) := \left(\frac{g1(k)}{g2(k)}\right)$$ $\quad T(k) = 0.8$ $\quad T(k)^2 = 0.64$