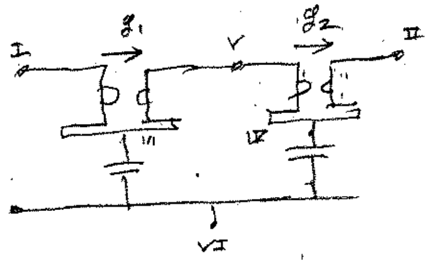
EE610 – Y(s)

10/13/05
p. 1



for g_1: $i_I = g_1(v_V - v_{III})$
$i_V = -g_1(v_I - v_{III})$
$i_{III} = -i_I - i_V$

for g_2: $i_V = g_2(v_{II} - v_{IV})$
$i_{II} = -g_2(v_V - v_{IV})$
$i_{IV} = -i_V - i_{II}$

for C_1: $i_{III} = -i_{VI} = sC_1(v_{III} - v_{VI})$
for C_2: $i_{IV} = -i_{VI} = sC_2(v_{IV} - v_{VI})$

$$\therefore\ Y_{indef} = \begin{array}{c} \begin{matrix} v_I & v_{II} & v_{III} & v_{IV} & v_V & v_{VI} \end{matrix} \\ \begin{bmatrix} & & -g_1 & & g_1 & \\ & & & g_2 & -g_2 & \\ g_1 & & sC_1 & & -g_1 & -sC_1 \\ & -g_2 & & sC_2 & g_2 & -sC_2 \\ -g_1 & g_2 & g_1 & -g_2 & & \\ & & -sC_1 & -sC_2 & & sC_1+sC_2 \end{bmatrix} \end{array}$$

gnd node VI $= Y_{def} = \begin{bmatrix} 0 & 0 & -g_1 & 0 & g_1 \\ 0 & 0 & 0 & g_2 & -g_2 \\ g_1 & 0 & sC_1 & 0 & -g_1 \\ 0 & -g_2 & 0 & sC_2 & g_2 \\ -g_1 & g_2 & g_1 & -g_2 & 0 \end{bmatrix}$

Eliminate nodes III, IV, V at one time to get $Y(s) = Y_{2\text{-port}}$

$$Y(s) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -g_1 & 0 & g_1 \\ 0 & g_2 & -g_2 \end{bmatrix} \begin{bmatrix} sC_1 & 0 & -g_1 \\ 0 & sC_2 & g_2 \\ g_1 & -g_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} g_1 & 0 \\ 0 & -g_2 \\ -g_1 & g_2 \end{bmatrix}$$

But $\begin{bmatrix} sC_1 & 0 & -g_1 \\ 0 & sC_2 & g_2 \\ g_1 & -g_2 & 0 \end{bmatrix}^{-1} \Rightarrow \Delta = sC_1 \begin{vmatrix} sC_2 & g_2 \\ -g_2 & 0 \end{vmatrix} + g_1 \begin{vmatrix} 0 & -g_1 \\ sC_2 & -g_2 \end{vmatrix} = sC_1 g_2^2 + sC_2 g_1^2$

$= s(C_1 g_2^2 + C_2 g_1^2)$

$\Delta_{11} = g_2^2$, $\Delta_{21} = g_1 g_2$ $\Delta_{31} = sC_2 g_1$

$\Delta_{12} = g_1 g_2$ $\Delta_{22} = g_1^2$ $\Delta_{32} = -sC_1 g_2$

$\Delta_{13} = -sC_2 g_1$ $\Delta_{23} = sC_1 g_2$ $\Delta_{33} = s^2 C_1 C_2$

$$\therefore\ Y(s) = \begin{bmatrix} g_1 & 0 & -g_1 \\ 0 & -g_2 & g_2 \end{bmatrix} \begin{bmatrix} g_2^2 & g_1 g_2 & sC_2 g_1 \\ g_1 g_2 & g_1^2 & -sC_1 g_2 \\ -sC_2 g_1 & sC_1 g_2 & s^2 C_1 C_2 \end{bmatrix} \begin{bmatrix} g_1 & 0 \\ 0 & -g_2 \\ -g_1 & g_2 \end{bmatrix} \frac{1}{s(C_1 g_2^2 + C_2 g_1^2)}$$

$$= \begin{bmatrix} g_1 g_2^2 + sC_2 g_1^2 & g_1^2 g_2 - sC_1 g_1 g_2 & sC_2 g_1^2 - s^2 C_1 C_2 g_1 \\ -g_1 g_2^2 - sC_2 g_1 g_2 & -g_2 g_1^2 + sC_1 g_2^2 & sC_1 g_2^2 + s^2 C_1 C_2 g_2 \end{bmatrix} \begin{bmatrix} g_1 & 0 \\ 0 & -g_2 \\ -g_1 & g_2 \end{bmatrix} \cdot \frac{1}{s(C_1 g_2^2 + C_2 g_1^2)}$$

$$= \frac{1}{s(C_1 g_2^2 + C_2 g_1^2)} \begin{bmatrix} g_1^2 g_2^2 + s\cancel{C_2} g_1^3 - s\cancel{C_2} g_1^3 + s^2 C_1 C_2 g_1^2 & -g_1^2 g_2^2 + sC_1 g_1 g_2^2 + sC_2 g_1^2 g_2 - s^2 C_1 C_2 g \\ -g_1^2 g_2^2 - sC_2 g_1^2 g_2 - sC_1 g_1 g_2^2 - s^2 C_1 C_2 g_1 g_2 & g_1^2 g_2^2 - s\cancel{C_1} g_2^3 + s\cancel{C_1} g_2^3 + s^2 C_1 C_2 g_2^2 \end{bmatrix}$$

$$Y(s) = \frac{1}{s(c_1 g_2^2 + c_2 g_1^2)} \begin{bmatrix} s^2 c_1 c_2 g_1^2 + g_1^2 g_2^2 & -s^2 c_1 c_2 g_1 g_2 + s(c_1 g_1 g_2^2 + c_2 g_1^2 g_2) - g_1^2 g_2^2 \\ -s^2 c_1 c_2 g_1 g_2 - s(c_1 g_1 g_2^2 + c_2 g_1^2 g_2) - g_1^2 g_2^2 & s^2 c_1 c_2 g_2^2 + g_1^2 g_2^2 \end{bmatrix}$$

$$= \frac{s\, c_1 c_2 g_1^2}{c_1 g_2^2 + c_2 g_1^2} \begin{bmatrix} 1 & -g_2/g_1 \\ -g_2/g_1 & (g_2/g_1)^2 \end{bmatrix} + \frac{c_1 g_1 g_2^2 + c_2 g_1^2 g_2}{c_1 g_2^2 + c_2 g_1^2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \frac{g_1^2 g_2^2}{c_1 g_2^2 + c_2 g_1^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$