10/19/05

Lossless synthesis (1x1 matrices)

$z(s) = \frac{1}{y(s)}$ and $y(s)$ are lossless together

Ex: $$y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+4)} = -y(-s)$$

here $z(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+2)}$ has a pole at $s=0$

1st Foster: partial fraction expansion of $z(s)$

$$z(s) = \frac{k_0}{s} + \frac{2k_1 s}{s^2+2} + k_\infty s \qquad k\text{'s} > 0$$

$$\frac{(s^2+1)(s^2+4)}{s(s^2+2)} =$$

$$k_0 = \cancel{s}\left(\frac{(s^2+1)(s^2+4)}{\cancel{s}(s^2+2)} - \frac{2k_1 s^2}{s^2+2} - k_\infty s^2\right)\Bigg|_{s=0} = \frac{(0+1)(0+4)}{(0+2)}$$

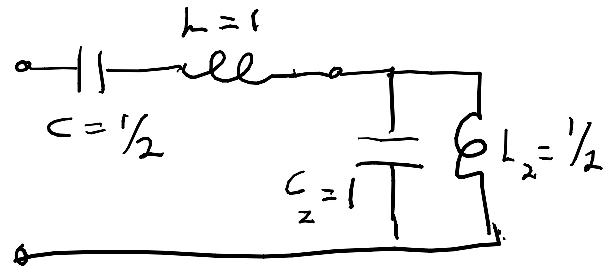
$$= 2$$

$$2k_1 = \cancel{\frac{s^2+2}{s}}\left(\frac{(s^2+1)(s^2+4)}{s\cancel{(s^2+2)}}\right) - \frac{k_0}{s} \times \frac{s^2+2}{s} - k_\infty s\left(\frac{s^2+2}{s}\right)\Bigg|_{s^2=-2}$$

$$= \frac{(s^2+1)(s^2+4)}{s^2}\Bigg|_{s^2=-2} = \frac{(-2+1)(-2+4)}{-2} = +1$$

$$k_\infty \sim \frac{(s^2)(s^2)}{s(s^2)} \cdot \frac{1}{s} = 1$$

$$z(s) = \frac{2}{s} + s + \frac{s}{s^2+2} \rightarrow$$



1st Foster

$$\begin{matrix} \| \\ z\left\{ \frac{1}{} \right. \\ y\left\{ s + \frac{1}{s/2} \right. \end{matrix}$$

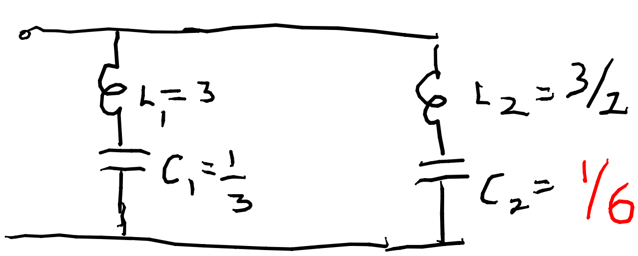
2nd Foster: partial fraction expansion of $y(s)$

$$y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+4)} = \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+4}$$

$$2k_1 = \left.\frac{s^2+2}{s^2+4}\right|_{s^2=-1} = \frac{2-1}{4-1} = 1/3$$

$$2k^2 = \left.\frac{s^2+2}{s^2+1}\right|_{s^2=-4} = \frac{-2}{-3} = 2/3$$

$$y(s) = \frac{1}{z(s)} = \frac{\frac{1}{3}s}{s^2+1} + \frac{\frac{2}{3}s}{s^2+4} = \frac{1}{3s + \frac{1}{\frac{1}{3}s}} + \frac{1}{\frac{3}{2}s + \frac{4}{\frac{2}{3}s}}$$



2nd Foster

1st Cauer: a continued fraction expansion about $s = \infty$; we remove poles

$$Y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+4)} = \frac{s^3+2s}{s^4+5s^2+4}$$

$$Z(s) = \frac{1}{Y(s)} = \frac{s^4+5s^2+4\cancel{s}}{s^3+2s} = s + \frac{3s^2+4}{s^3+2s} = s + \cfrac{1}{\cfrac{s^3+2s}{3s^2+4}}$$

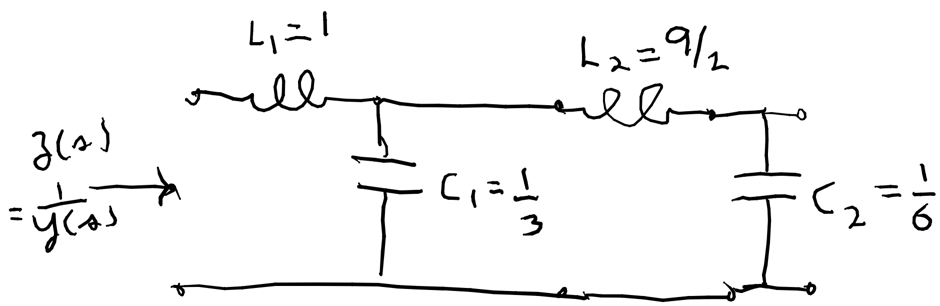
$$\begin{array}{r|l} & s \\ s^3+2s & s^4+5s^2+4 \\ & s^4+2s^2 \\ \hline & 3s^2+4 \end{array}$$

$$\begin{array}{r|l} & \frac{1}{3}s \\ 3s^2+4 & s^3+2s \\ & s^3+\frac{4}{3}s \\ \hline & \frac{2}{3}s \end{array}$$

$$\begin{array}{r|l} & \frac{9}{2}s \\ \frac{2}{3}s & 3s^2+4 \\ & 3s^2 \\ \hline & 4 \end{array}$$

$$\begin{array}{r|l} & \frac{2}{12}s \\ 4 & \frac{2}{3}s \end{array}$$

$$Y(s) = \cfrac{1}{s + \cfrac{1}{\frac{1}{3}s + \cfrac{1}{\frac{9}{2}s + \cfrac{1}{\frac{1}{6}s}}}}$$



2nd Cauer: continued fraction expansion about $s = 0$

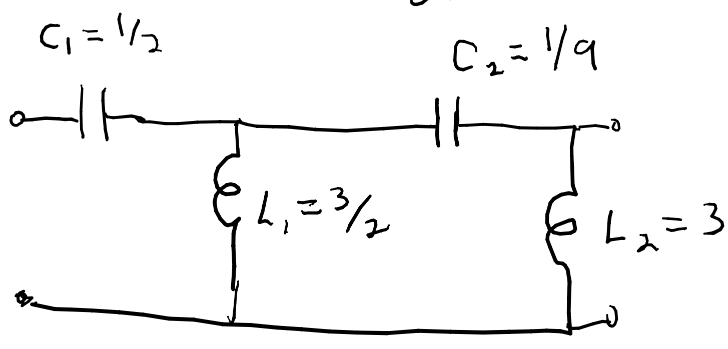
$$y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+4)} = \frac{s^3+2s}{s^4+5s^2+4}$$

$z(s) = \dfrac{2}{s}$ + remainder (no pole at zero)

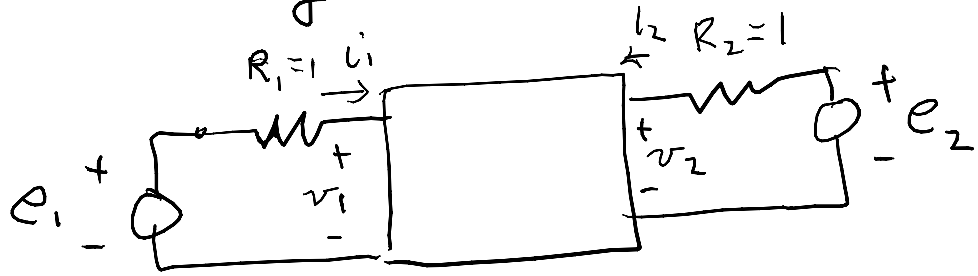
(i.e. its $y_1(s)$ has a pole at $s = 0$)

$$
\begin{array}{l}
\dfrac{4}{2s} \\
2s+s^3\;\overline{)\;4+5s^2+s^4} \\
\underline{4+2s^2} \\
3s^2+s^4\;\overline{)\;2s+s^3}\;\;\dfrac{2}{3s} \\
\underline{2s+\tfrac{2}{3}s^3} \\
\tfrac{1}{3}s^3\;\overline{)\;3s^2+s^4}\;\;\dfrac{9}{s} \\
\phantom{2s+s^3\;)\;4+3s^2+s^4\;)\;2s+\tfrac{1}{3}s^3\;)\;}\underline{3s^2} \\
\phantom{2s+s^3\;)\;4+3s^2+s^4\;)\;2s+\tfrac{1}{3}s^3\;)\;3s^2+}s^4\;\overline{)\;\tfrac{1}{3}s^3}\;\;\dfrac{1}{3s}
\end{array}
$$

$$y(s) = \cfrac{1}{\cfrac{2}{s} + \cfrac{1}{\cfrac{2}{3s} + \cfrac{1}{\cfrac{9}{s} + \cfrac{1}{\cfrac{1}{3s} + 0}}}}$$



Scattering matrix



$$e = 2v^i = v + Ri$$

$$2v^r = v - Ri$$

$$v^r = S v^i$$

$R = 1_2$

v^i = incident
v^r = reflected

S = scattering matrix

Use: Let $e_2 = 0 \Rightarrow v_2 = -i_2$

$$v_2^i = 0 \Rightarrow 2v_2^r = 2v_2$$

$$e_1 = 2v^i \Rightarrow S_{21} = \left.\frac{v_2^r}{v_1^i}\right|_{v_2^i = 0} = \frac{v_2}{e_1/2} = 2\frac{v_2}{e_1}$$

Know $Av = Bi$; $Y = B^{-1}A$

But $v = v^i + v^r$, $i = v^i - v^r$ $\Rightarrow$ $Av^i + Av^r = Bv^i - Bv^r$

$$(A+B)v^r = (B-A)v^i$$

$$\therefore S = (B+A)^{-1}(B-A) = (1_m + B^{-1}A)^{-1}B^{-1}B(1_m - B^{-1}A)$$

$$= (1_m + Y)^{-1}(1_m - Y)$$