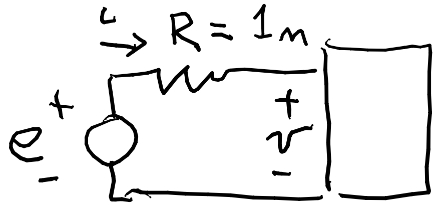
10/17/05

p. 342 lossless synthesis
(of reactance functions)



$e \in \mathcal{L}_2$ = Lebesgue

$$\|e\|^2_{\mathcal{L}_2} = \int_{-\infty}^{\infty} e^{T*}(t)\, e(t)\, dt < \infty$$

$$e = v + i$$

$$e^{T*}e = (v+i)^{T*}(v+i) = v^{T*}v + i^{T*}i + v^{T*}i + i^{T*}v$$

$$\|e\|^2_{\mathcal{L}_2} = \|v\|^2 + \|i\|^2 + 2\,\mathrm{Re}\int_{-\infty}^{\infty} v^{T*}(t)\, i(t)\, dt \quad \text{if } \mathrm{Re}\int_{-\infty}^{\infty} v^{T*}i \geq 0$$

(passive)

then $v, i \in \mathcal{L}_2$

Lossless means $= 0$

$\Rightarrow$ energy is disipated in terminating R

Lossless $\Rightarrow$ $Y(j\omega) + Y^{T*}(j\omega) = 0_n$

if $Y(s)$ are rational with real coefficients then

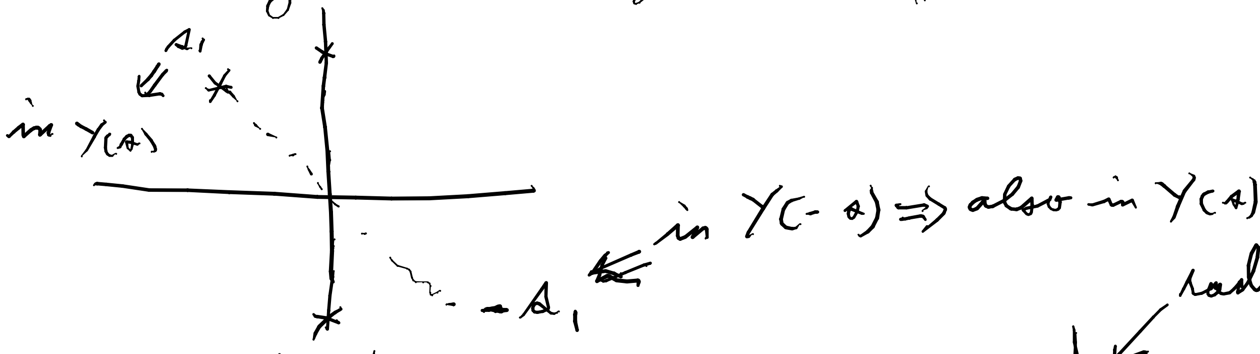
$*$ goes onto $j\omega$ $\Rightarrow$ $Y(j\omega) + Y^{T}(-j\omega) = 0_n$

if rational can extend to $s = \sigma + j\omega$ by setting $\omega = s/j$ valid in all $s = \sigma + j\omega$ except at poles.

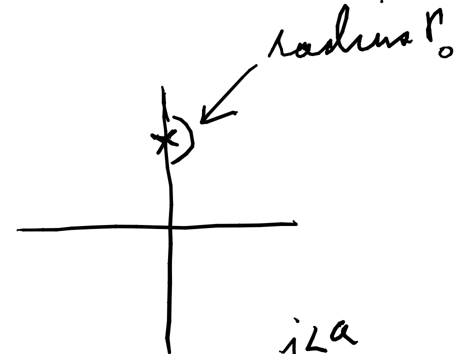
$$Y(s) + Y^{T}(-s) = 0_n \quad \text{(except possibly at poles)}$$

$$Y(s) = -Y^{T}(-s) = -Y^{T}_{*}(s) \longleftarrow \text{Hurwitz conjugate}$$

all poles of s are on the $j\omega$ axis, if positive-real



$\therefore$ no poles in $\sigma > 0$ or $\sigma < 0$



For $n = 1$, $y(s) = -y(-s)$

near a pole $y(s) \sim \dfrac{a}{(s + j\omega_0)^k}$

$$\mathrm{Re} \sim \mathrm{Re}\left(\frac{a}{(s + j\omega_0)^k}\right) = \mathrm{Re}\,\frac{|a| e^{j\angle a}}{r e^{j\angle(s - j\omega_0)k}}$$

$\angle a = 0$ & $k = 1$ if pos. real

$\Rightarrow$ poles on $j\omega$ axis are simple $(k=1)$ and residues (a) are positive (real)

Ex: $y(s) = \dfrac{2s}{s^2 + 4}$; $y(s) = -y(-s)$

$$= \frac{a_1}{s + j2} + \frac{a_1^*}{s - j2} \;;$$

$$a_1 = (s + j2)\left[\frac{2s}{s^2 + 4} - \frac{a_1^*}{s - j2}\right] = \frac{2s}{s - j2} + (s + j2)\left(\frac{-a_1^*}{s - j2}\right)$$

as $a_1 =$ const set $s = -j2$

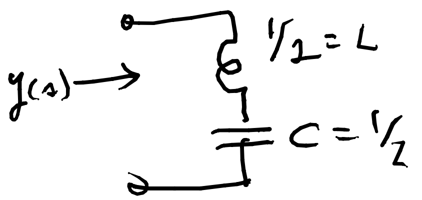
$$= \frac{-j4}{2(-j2)} + 0$$

$$= 1$$

$$y(s) = \frac{1}{s+j2} + \frac{1}{s-j2} = \frac{2s}{s^2+4}$$

$\therefore$ $y(s)$ is a lossless PR function

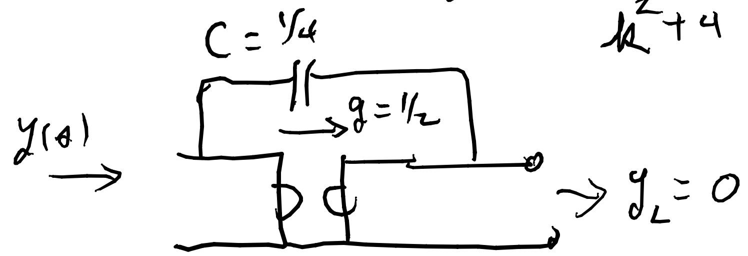
$$y(s) = \frac{2s}{s^2+4} = \frac{1}{\frac{s^2}{2s}+\frac{4}{2s}} = \frac{1}{\frac{s}{2}+\frac{2}{s}} = \frac{1}{\text{impedance}}$$



for a Richards' function synthesis can use any k as $y(s)$ has an identically zero even part.

Ex: $k = 2$ $\quad y(k) = \frac{2k}{k^2+4} = \frac{4}{8} = 1/2$, $\quad g = y(k) = 1/2$

$$C = \frac{y(k)}{k} = 1/4$$



Ex: If $y(s)$ is PR then so is $\frac{1}{y(s)} = z(s)$

$$y(s) = \frac{2s^2}{s^2+4}$$; not lossless as not odd

$$y(s) \neq -y(-s) = \frac{-2s^2}{s^2+4}$$

$$z(s) = \frac{s^2+4}{2s^2}$$; has a double pole at $s=0$ so can not be PR

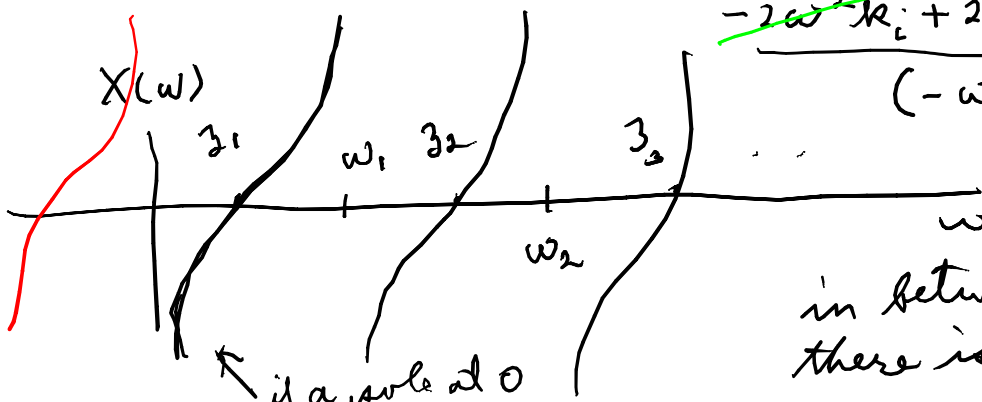
$\therefore$ no passive circuit exists for this $y(s)$

If $y(s)$ is lossless, poles and zeros alternate on the $j\omega$ axis:

$$y(s) = k_\infty s + \frac{k_0}{s} + \sum_{i=1}^{m} \frac{2k_i s}{s^2 + \omega_{k_i}^2} \quad \text{for } k\text{'s} \geq 0$$

$$y(j\omega) = jk_\infty \omega - j\frac{k_0}{\omega} + \sum_{i=1}^{m} \frac{j\,2k_i\,\omega}{-\omega^2 + \omega_{k_i}^2} = jX(\omega)$$

$$\frac{dX(\omega)}{d\omega} = k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1}^{m} \underbrace{\frac{2k_i}{(-\omega^2 + \omega_{k_i}^2)} - \frac{2k_i\omega(-2\omega)}{(-\omega^2+\omega_{k_i}^2)^2}}_{\frac{\cancel{-2\omega^2 k_i} + 2k_i\omega_k^2 + 2\cancel{4}k_i\omega^2}{(-\omega^2+\omega_{k_i}^2)^2}} \geq 0$$



in between each pole there is a zero

$X(\omega) = -X(-\omega)$ is the "reactance"

∴

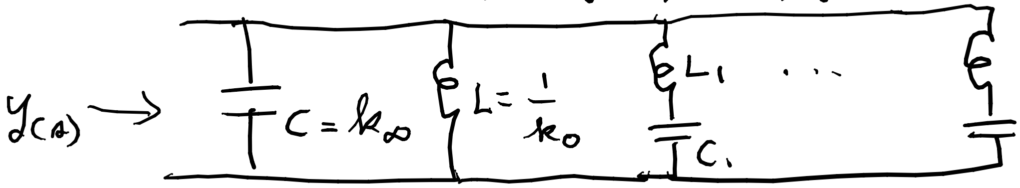
Ex: $y(s) = \dfrac{s(s^2+3)}{s^2+1}$ is PR & lossless

but $y(s) = \dfrac{s(s^2+3)}{s^2+5}$ is not as poles and zeros do not alternate

$y(s) = \dfrac{s(s^2+3)^2}{s^2+1}$ is not as $z(s) = 1/y$ has a double pole at $s = \pm j\sqrt{3}$

$Y(s)$ lossless (PR) yields

2nd Foster form = partial fraction expansion of $Y(s)$

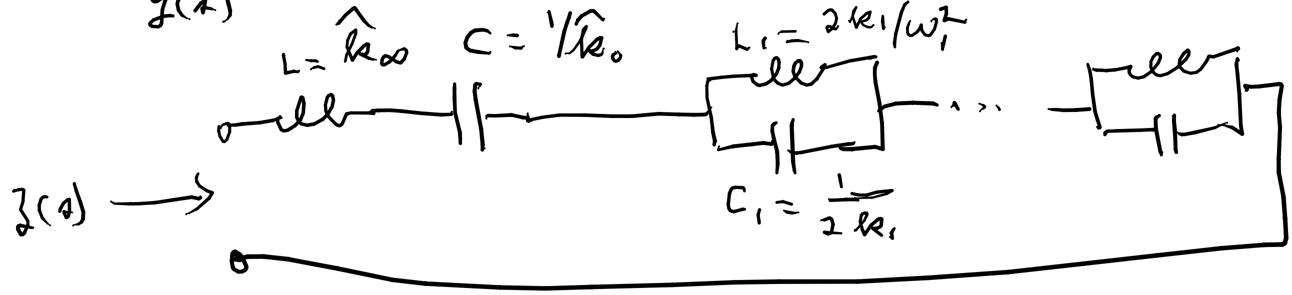


$$y_i(s) = \frac{2k_i s}{s^2 + \omega_{k_i}^2} = \frac{1}{\frac{s}{2k_i} + \frac{\omega_{k_i}^2}{2k_i s}}$$

$$C_i = \frac{2k_i}{\omega_{k_i}^2}, \quad L_i = \frac{1}{2k_i}$$

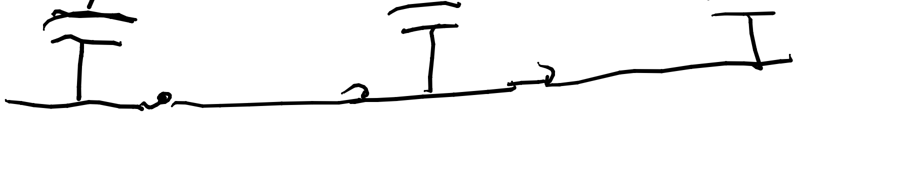
1st Foster $\Rightarrow$ partial fraction expansion of $Z(s)$

$$Z(s) = \frac{1}{Y(s)} = \hat{k}_\infty \cdot s + \frac{\hat{k}_0}{s} + \sum_{i=1}^{\hat{m}} \frac{2\hat{k}_i s}{s^2 + \hat{\omega}_i^2}$$



2 other Canonical forms are those due Cauer

1st; removes poles at ∞, makes a continued fraction expansion:



gives this ladder structure

removing poles at zero gives 2nd Cauer