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Positive Real Matrices $\Leftarrow$ passive circuits

$Y(s)$, $s = \sigma + j\omega$ = complex variable

Def: 1) $Y(s)$ is real valued for $\sigma > 0$ and s real (means real components)

2) $Y(s)$ has no singularities in $\sigma > 0$ (stability)

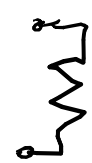
3) $Y(s) + Y^{T*}(s)$ is positive semidefinite in $\sigma > 0$ (all components are passive)

Ex: #1) $Y(s) = G$ (G = constant) (number)

1) ok if G is real

2) ok no singularities

3) $G + G \geq 0 \Rightarrow G \geq 0$


$R = 1/G$

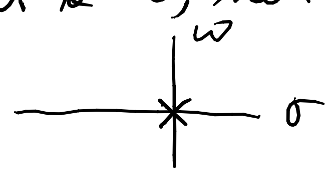
$\therefore$ $Y(s) = G$ is positive-real if $G \geq 0$

#2) $Y(s) = \dfrac{1}{s} \Rightarrow$  $L = 1$

1) ok as coefficients are real; $Y(\sigma) = \dfrac{1}{\sigma}$

real for all $\sigma \neq 0$

2) a pole at $s = 0$, not in $\sigma > 0$

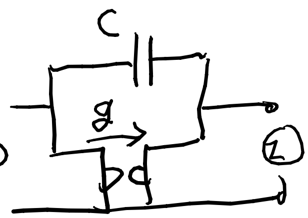


condition 3) $Y(s) + Y(s)^* = \dfrac{1}{\sigma + j\omega} + \dfrac{1}{\sigma - j\omega}$, $\sigma > 0$

$$= \frac{2\sigma}{|\sigma + j\omega|^2} > 0 \text{ in } \sigma > 0$$

$\therefore \; Y(s) = \dfrac{1}{s}$ is PR

#3



$$Y(s) = \begin{bmatrix} sC & -sC + g \\ -sC - g & sC \end{bmatrix}$$

1) if C & g are real $Y(\sigma)$ is real for $\sigma > 0$

2) has a pole at ∞, assume ∞ is on $j\omega$ axis so no pole in $\sigma > 0$

3) $Y(s) + Y^{T*}(s) = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix} + \begin{bmatrix} s^*C & -s^*C-g \\ -s^*C+g & s^*C \end{bmatrix}$

$$= C\begin{bmatrix} s + s^* & -s - s^* \\ -s - s^* & s + s^* \end{bmatrix} = 2C\begin{bmatrix} \sigma & -\sigma \\ -\sigma & \sigma \end{bmatrix}$$

a matrix $H = H^{*T}$ is positive semidefinite if for all complex vectors V, $V^{T*} H V \geq 0$

$$[V_1^* \; V_2^*]\left\{2C\begin{bmatrix} \sigma & -\sigma \\ -\sigma & \sigma \end{bmatrix}\right\}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 2C\,[V_1^* V_2^*]\begin{bmatrix} \sigma(V_1 - V_2) \\ -\sigma(V_1 - V_2) \end{bmatrix}$$

$$= 2C\sigma\,[\,V_1^*V_1 - V_1^*V_2 - V_2^*V_1 + V_2^*V_2\,]$$

Let $V_1 = V_{1r} + jV_{1i}$, $V_2 = V_{2r} + jV_{2i}$

$$V_1^* V_2 = (V_{1r} - jV_{1i})(V_{2r} + jV_{2i}) \Rightarrow \mathrm{Re}\, V_1^* V_2 = V_{1r}V_{2r} + V_{1i}V_{2i}$$

$$V_1^* V_1 - 2\,\mathrm{Re}(V_1^* V_2) + V_2^* V_2 = V_{1r}^2 + V_{1i}^2 - 2V_{1r}V_{2r} - 2V_{1i}V_{2i} + V_{2r}^2 + V_{2i}^2$$

$$= (V_{1r} - V_{2r})^2 + (V_{1i} - V_{2i})^2$$

$$V^{T*}(Y + Y^{T*})V = 2C\sigma\left[(V_{1r} - V_{2r})^2 + (V_{1i} - V_{2i})^2\right] \geq 0$$

it can be 0 when $V_1 = V_2$ (can be 0 for $V = \begin{bmatrix} V_1 \\ V_1 \end{bmatrix} \neq 0$ but can not be < 0 for $\sigma > 0$, $C > 0$)

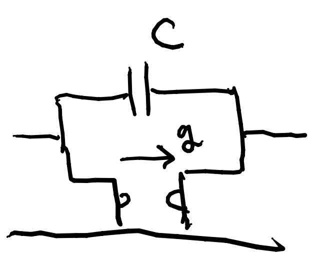
Here $Y(s) + Y^T(-s) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} sC & -sC + g \\ -sC - g & sC \end{bmatrix} + \begin{bmatrix} -sC & sC - g \\ sC + g & -sC \end{bmatrix}$$

$-s = s^*$ if $s = j\omega$, $s = \sigma + j\omega \Rightarrow s = j\omega$ then $-s = -j\omega = s^*$

If $Y(s) + Y^T(-s) = 0_n$ then this is the lossless condition

$\therefore$



is lossless if $C \geq 0$, g real (passive also under these conditions)

$$p(t) = v^T(t)\, i(t) \Rightarrow \mathcal{E}(\infty) = \int_{-\infty}^{\infty} v^T(t)\, i(t)\, dt$$

$$= \langle v, i \rangle_t \quad \text{well defined if } v, i \text{ in } \mathcal{L}_2$$

Ex: $e^{-2t} 1(t)$ is in $\mathcal{L}_2$

$$\int_{-\infty}^{\infty} \left(e^{-2t} 1(t)\, e^{-2t} 1(t)\right) dt = \int_0^{\infty} e^{-4t} dt = \frac{1}{-4} e^{-4t}\Big|_0^{\infty} = +\frac{1}{4} < \infty$$

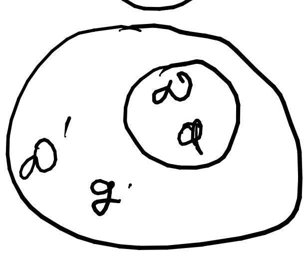
but $1(t)$ is not in $\mathcal{L}_2$ as $\int_0^{\infty} 1\, dt = \infty$

If we have $\mathcal{L}_2$ functions then we have Fourier transforms of them. ∴ use Parseval's theorem to go between time & frequency domain:

$$\langle g^{*T}, \varphi \rangle = \langle \mathcal{F}^{*T}[g], \mathcal{F}[\varphi] \rangle$$

$$\int_{-\infty}^{\infty} g^{*T}(t)\, \varphi(t)\, dt = \int_{-\infty}^{\infty} \mathcal{F}^{*T}[g](2\pi f) \cdot \mathcal{F}[\varphi](2\pi f)\, df$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{F}^{*}[g]^{T} \cdot \mathcal{F}[\varphi]\, d\omega$$



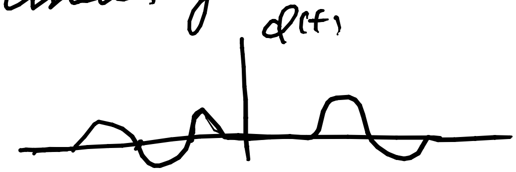
$\mathcal{D}$ = space of test functions

$\mathcal{D}'$ = dual space = space of "distributions"

= normal functions, impulses and all derivatives

Take $\mathcal{D}$ = set of infinitely differentiable functions then the dual space is the set of continuous linear functionals on $\mathcal{D}$

$\mathcal{D}'$



$$\int_{-\infty}^{\infty} g(t)\,\phi(t)\,dt = \langle g, \phi \rangle$$

Ex: $\delta(t)$ is in $\mathcal{D}'$ defined by $\langle \delta, \phi \rangle = \phi(0)$

$$= \int_{-\infty}^{\infty} \delta(t)\,\phi(t)\,dt = \phi(0)$$