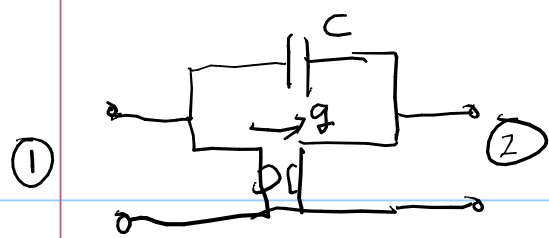
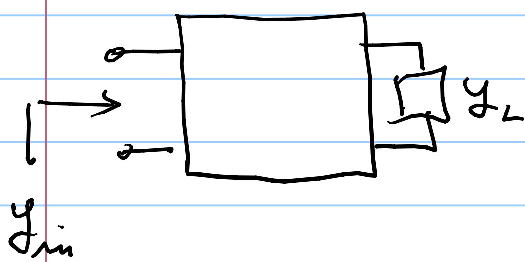
09/28/05



$$Y = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}; \quad \Delta_y = g^2$$



$$y_{in} = y_{11} - y_{12}(y_{22}+y_L)^{-1} y_{21}$$
$$= \frac{y_{11}y_{22} + y_{11}y_L - y_{12}y_{21}}{y_{22}+y_L}$$
$$= \frac{y_{11}y_L + \Delta_y}{y_{22}+y_L}$$

$$y_{in}y_{22} + y_{in}y_L = y_{11}y_L + \Delta_y$$

$$(y_{in} - y_{11})y_L = \Delta_y - y_{in}y_{22} \Rightarrow y_L = \frac{y_{in}y_{22} - \Delta_y}{y_{11} - y_{in}}$$

$$y_L(s) = \frac{sC \cdot y_{in}(s) - g^2}{sC - y_{in}(s)} = \frac{g^2\left(\frac{sC}{g^2} \cdot y_{in}(s) - 1\right)}{sC - y_{in}(s)}$$

go look at Richards' function, p. 361

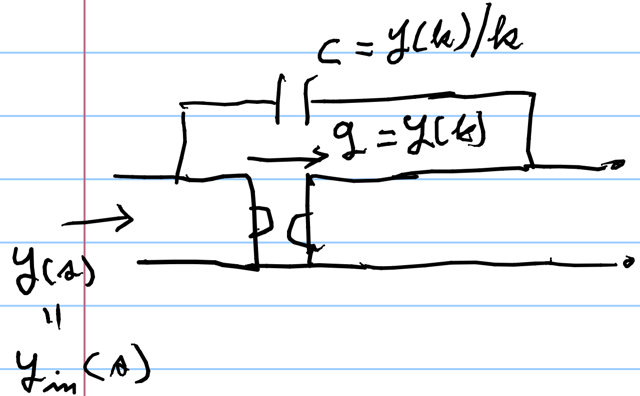
$$R_2(s) = G_R(s) = \frac{s\,y(s) - k\,y(k)}{s\,y(k) - k\,y(s)}$$
$$= \frac{k\,y(k)}{k}\left(\frac{\frac{s}{k\,y(k)} \cdot y(s) - 1}{\frac{s\,y(k)}{k} - y(s)}\right)$$

note $s = k$ is a zero of the numerator & the denominator (for rational $y(s)$)

let $y(s) = y_{in}(s)$, $C = y(k)/k$; $\frac{C}{g^2} = \frac{1}{k\,y(k)}$

$$g^2 = C \cdot k\, y(k) = \frac{y(k)}{k} \cdot k\, y(k) = y^2(k)$$

$$g = \pm\, y(k) \qquad \Rightarrow \text{ gives } y_L(s) = y(k)\, G_R(s)$$



$$y_L(s) = y(k)\, G_R(s)$$
$$= y(k)\left(\frac{s\, y(s) - k\, y(k)}{s\, y(k) - k\, y(s)}\right)$$

look at $s = -k$

gives (numerator) $-k\, y(-k) - k\, y(k) = 0$ if $y(k) + y(-k) = 0$

i.e. a factor $s + k$ if k is a zero of $Ev\, y(s)$

denominator

$-k\, y(k) - k\, y(-k) = 0$ if k is a zero of $Ev\, y(s)$

if $k \neq 0$ or $y(k) \neq 0$

Ex: $y(s) = \dfrac{3^{2}s + 2}{s+2}$; $2Ev\, y(s) = y(s) + y(-s) = y + y_*$ Hurwitz conjugate

$$= \frac{3^{2}s + 2}{s+2} + \frac{-3^{2}s + 2}{-s+2}$$

$$= 2\,\frac{(-3^{2}s^2 + 4)}{(-s^2 + 4)}$$

zeros at $-3^{2}s^2 + 4 = 0$

$s = \pm \dfrac{2}{\cancel{\sqrt{3}}\,3} = \pm \dfrac{2}{3}$ choose $k = 2/3$

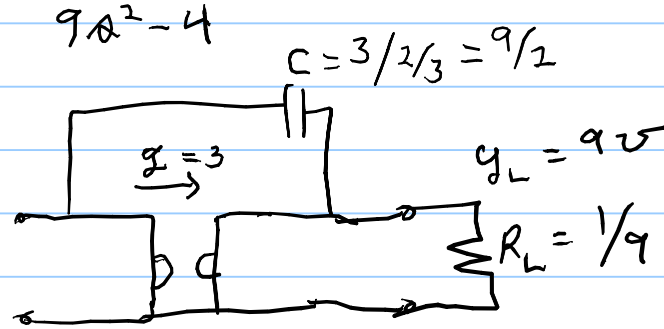
$$y(k) = y\left(\tfrac{2}{3}\right) = \frac{9 \times 2/3 + 2}{(2/3) + 2} = \frac{6+2}{8/3} = 3 = y(k)$$

$$G_R(s) = \frac{s\,y(s) - k\,y(k)}{s\,y(k) - k\,y(s)} = \frac{s\,\frac{9s+2}{s+2} - \frac{2}{3}\times 3}{s \cdot 3 - \frac{2}{3}\,\frac{9s+2}{s+2}} = \frac{3s(9s+2) - 6(s+2)}{9s(s+2) - 2(9s+2)}$$

$$= \frac{27s^2 - 12}{9s^2 - 4} = \frac{3(9s^2 - 4)}{9s^2 - 4} = 3$$

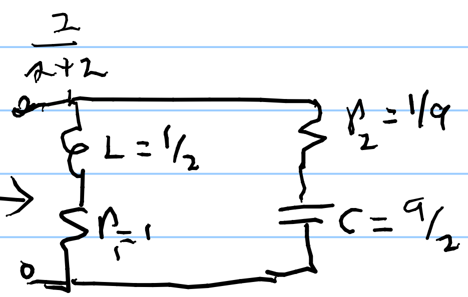
$$y_L(s) = 3 \cdot 3 = 9$$

$$y(s) = \frac{9s+2}{s+2} \rightarrow$$



$$= \frac{9s}{s+2} + \frac{2}{s+2} = \frac{1}{\frac{s+2}{9s}} + \frac{1}{\frac{s}{2}+1} = \frac{1}{\frac{1}{9} + \frac{1}{\frac{9}{2}s}} + \frac{1}{\frac{s}{2}+1}$$

$y(s) \rightarrow$



uses 2 reactive elements (L & C)s & 2 resistors

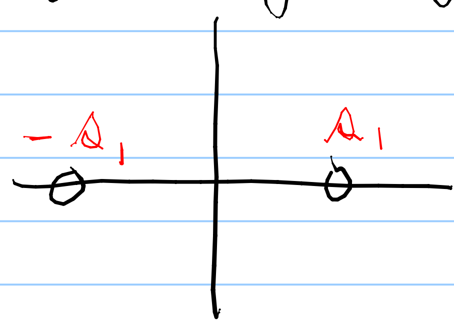
Can repeat above on $y_L(s)$, if use k = zero of the even parts, decrease degree at each step.

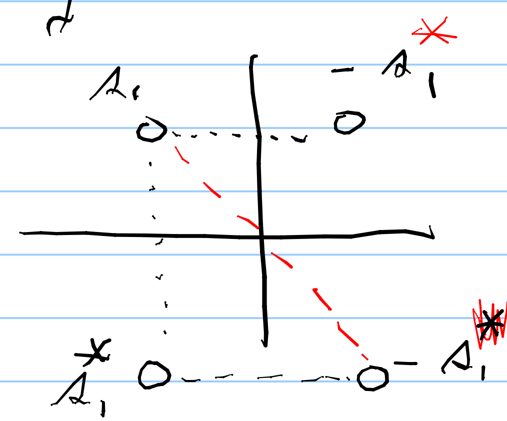
$Ev\, y(s)$ zeros. $\quad Ev(y(s)) = Ev\, y(-s)$

so if s_1 is a zero of $Ev\, y(s)$ then $-s_1$ is a zero

if s_1 is complex then s_1^* is also a zero due to $y(s)$ having real coefficients, i.e. if

if $y(s)$ is rational with real coefficients then the zeros of $Ev\, y(s)$ they look like





call quadrantal symmetry

$$G_R(s) = \frac{s\,y(s) - k\,y(k)}{s\,y(k) - k\,y(s)}$$

if k is complex then this does not have real coefficients if $y(s)$ does

if repeat at $s = k^* = k_2$ & k is a zero of $\mathrm{Ev}\, y(s)$ then no more complex coefficients on the second extraction.